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Close Encounters with Large Asteroids in the Next 50 Years

M. KUZMANOSKI

Institute of Astronomy, Studentski trg 16, 11000 Belgrade, Yugoslavia

AND

Z. KNEŽEVIĆ

Astronomical Observatory, Volgina 7, 11050 Belgrade, Yugoslavia

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A combined method of searching for asteroid close encounters has been described and applied for detection of close encounters with largest asteroids in a period of 50 years starting from 1991. The method consists of a multistep selection procedure and an accurate numerical integration and provides the parameters of close encounters with high reliability. We compiled a list of 208 close encounters with asteroids larger than 100 km in diameter occuring at distances less than 0.01 AU. Expected mutual perturbing effects have been computed for the most interesting pairs. Measurable effects, enabling an asteroid mass determination, have been found for three of the examined pairs (1, 1393; 4, 2831; 10, 2061), whereas smaller effects, requiring an improved observational accuracy, better orbital fits, and more reliable ephemerides, were detected in several other cases.

1. INTRODUCTION

A lot of effort has been put so far into attempts to determine the asteroid masses, but the results are few and unreliable (see Schubart and Matson, 1979, for the results of some earlier attempts, and Landgraf, 1988, and Hoffmann, 1989a, for the more recent ones). Due to the smallness of the asteroids the effects they impose on the motions of the other Solar System bodies are also very small, usually even too small to be accurately measured, with the only exception occurring in the case of close encounters between the asteroids themselves (e.g., Scholl *et al.* 1987, Goffin 1991, Williams 1992). Although some results might be obtained in the framework of the Hipparcos mission (Bec-Borsenberger 1991), the dedicated space missions which would provide the accurate asteroid masses do not seem to be a priority for the near future. Thus, for the time being the close encounters remain the only available and potentially fruitful source of data necessary for the asteroid mass determination, which can be exploited even by means of the standard ground based observational techniques. On the other hand, the close encounters, in particular the very close, long-lasting ones and those involving larger bodies for which the mutual perturbations become measurable, are neither frequent nor easily predictable, and a careful and comprehensive study is necessary even for their exact locations in space and time.

A whole range of different methods has been developed in order to determine whether and when two asteroids will approach each other to such a distance that their mutual perturbations become significant. First attempts, dating back to 19th century, and even some current undertakings, dealt with a concept of the so-called *asteroid proximities*, representing essentially the minimum distances between the bodies (see Michkovitch, 1974, for a survey of the old methods and Simovljevitch, 1977, for the more contemporary ones). All these methods were based on successive approximations and employed various analytical, graphical, or combined procedures to determine the positions of asteroids in the proximity. Another approach, using the preselection of the *quasicomplanar* asteroids (asteroids with small mutual inclinations), has been developed by Lazović (1967, 1974, 1978) and applied by Lazović and Kuzmanoski (1974, 1978, 1979) to find the minimum possible distances between pairs of asteroid orbits; the perturbed motions and changes of the orbits in proximities were considered by Lazović and Kuzmanoski (1978) and Simovljevitch (1979). Hoffmann (1989b) used the same strategy, but employed different criteria for the selection—he considered asteroids with nearly equal semimajor axes and low eccentricities; analyzing the efficiency of the orbit-changing encounters in a period of 8 years, he found only one encounter close enough to enable

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the mass determination of an asteroid by astrometric techniques. Davis and Bender (1977) compiled a list of pairs that, due to the commensurability of the revolution periods of the involved objects, have periodically repeating close encounters accumulating the perturbing effects, which thus become large enough to be measured. The problem with this approach is that, in most cases, one has to wait for quite a long time to get a meaningful result. Finally, quite recently, Yoshikawa and Nakamura (1992) used a super computer to calculate all the "near-misses" among about 4500 numbered asteroids for almost 100 years into the future; however, the results of their study published so far pertain to the statistics of the asteroid close encounters and to the estimate of the corresponding collisional probability. Note, in addition, that Farinella and Davis (1992) developed a method for computation of asteroid collisional probabilities, which can also be applied to derive the frequency of asteroidal close encounters and the magnitudes of mutual perturbations among asteroids.

In the present paper we employed a complex, combined method based on a multistep selection procedure and numerical integration to search for the asteroid close encounters for a period of 50 years into the future. We were particularly interested in finding whether it is possible to reveal some close encounters giving rise to measurable perturbations useful for the mass determinations; i.e., what are the most interesting cases, and how large are the expected effects? Note that a similar attempt has already been made by Kuzmanoski (1992a, b), but covering a different time span and employing a different procedure. The selection method is described in Section 2 of this paper and results are given and discussed in Section 3.

2. METHOD OF SELECTION

As explained by Hoffmann (1989a), the astrometric method for determination of asteroid masses consists of determining the deviations of the asteroid observed positions from the ephemerides, which can be attributed to the perturbation by another asteroid. Due to the limited accuracy of the groundbased observations, orbital fits, and resulting ephemerides, only residuals in excess of approximately $1''$, resulting from the change of orbital elements during the close encounter, can be reliably measured with no special effort and straightforwardly employed for the determination of the mass. On the other hand, most of the asteroid close encounters occur at distances and relative velocities enabling only marginal interaction of the involved bodies. Thus, despite of the great number of actual close encounters among asteroids (see Yoshikawa and Nakamura 1992, for the statistics), the choice of the usable ones reduces mainly to those with the largest bodies. This fact served as a primary criterion applied by most of the aforementioned authors in selecting the bodies for which the close encounters have been sought. The other criteria (limiting distance and velocity, time spans covered by the search, and numerical integration, etc.) were set more or less arbitrarily (usually in accordance with the ambition of the authors to make a more or less complete survey), and they vary from search to search. Since our primary goal in this paper was to identify and analyze just the close encounters which can provide opportunities for the asteroids mass determination and to reduce the huge amount of computations, we tried here to be at once more comprehensive and more restrictive than the others. Hence we covered the time span of 50 years, but considered only the close encounters with bodies larger than 100 km in diameter and occurring at mutual distances less than 0.01 AU. Thus, we might have lost some interesting encounters with the very largest bodies (as the one occurring on 1996 June 16 among 4 Vesta and 17 Thetis, which should approach one another to a distance of 0.019 AU with a relative velocity of only 1.18 km/sec; D. Bender, private communication), but we eliminated a great number of encounters among smaller bodies occuring at greater distances which produce no measurable effects. For a comparison, let us mention that Davis and Bender covered a time span of 20 years, considering approaches to within 0.1 AU (in their case this is justified due to a particular choice of their pairs); Hoffmann covered only 8 years, searching for encounters at opposition with distance differences below 0.05 AU and with angular separation of less than 3°, but including those among very small bodies; Williams covered the complete period from the discovery of the involved bodies, but considered only approaches to within 0.1 AU of the largest asteroid 1 Ceres.

The critical point of any encounter search is how to efficiently and accurately perform an as-complete-as-possible selection of the asteroids which, within a given time span, will have what, according to some criteria, one considers a close approach. Here, as already mentioned, we applied for this purpose and using the above criteria a multistep procedure which consisted of the following: (i) first, by means of a simple geometrical consideration, we find all the pairs of asteroids whose osculating orbits, at a certain point or in a certain region, are close enough to enable a real approach of these bodies; (ii) then, by using two-body dynamics, we check whether for a given pair such an approach might occur within the given time span; (iii) finally, we perform a numerical integration of the two orbits in the framework of a simplified dynamic model (with four outer perturbing planets only) in order to determine more precisely parameters of the close encounter (approximate epoch, distance, relative velocity, location, etc.) and make a definite choice. Each of these steps reduces significantly the number of the cases to

TABLE I
Summary of the Selection Procedure; Close Encounters with 1 Ceres

Ast.1	Ast.2	Epoch[JD]	$\rho_{orb}[AU]$	$\Delta M[^\circ]$	$\rho[AU]$
1	57	2457761.1	0.04228	0.3	0.04556
1	142	2457953.7	0.02496	-0.8	0.05109
1	251	2457685.7	0.01546	0.1	0.02892
1	306	2456149.5	0.04084	-0.1	0.04480
1	324	2466489.7	0.01102	-0.3	0.20405
1	341	2449679.4	0.03491	0.1	0.04214
1	348	2462704.1	0.01575	-0.5	0.13707
1	389	2449569.3	0.03608	0.3	0.03996
1	391	2450126.9	0.04924	-0.9	0.05244
1	492	2463767.4	0.00306	-0.1	0.44185
1	631	2451101.1	0.01807	0.0	0.02204
1	668	2461221.8	0.03234	-0.7	0.12918
1	756	2460210.5	0.04886	-0.5	0.16774
1	786	2463926.6	0.02389	-0.6	0.98069
1	797	2454543.3	0.03330	-0.6	0.03931
1	934	2451368.9	0.01127	0.1	0.04499
1	957	2457858.9	0.02316	0.8	0.12320
1	1210	2457495.6	0.02721	0.3	0.12113
1	1253	2449561.2	0.02546	0.5	0.04305
1	1306	2465444.9	0.02495	-0.4	0.06929
1	1364	2465956.4	0.01235	-0.3	0.08061
1	1393	2459716.2	0.00116	-0.9	0.00754
1	1691	2462942.8	0.02387	-1.0	0.65716
1	1778	2456977.8	0.00135	-0.1	0.36995
1	1854	2458594.5	0.02463	-0.1	0.11303
1	2279	2466262.7	0.02385	-0.5	0.13999
1	2306	2459579.5	0.02147	0.4	0.09324
1	2377	2449522.4	0.03096	-0.7	0.04657
1	2398	2455070.3	0.02744	0.9	0.09304
1	2475	2449424.7	0.03092	-0.1	0.03142
1	2514	2456320.2	0.04024	-0.9	0.06118
1	2727	2461992.7	0.02854	-0.7	0.09470
1	2775	2460340.5	0.02809	-0.9	0.04816
1	2796	2459316.6	0.03432	-0.7	0.10965
1	2884	2466376.2	0.01298	0.5	0.87679
1	2898	2463321.4	0.04906	0.1	0.06460
1	2930	2453753.9	0.04793	0.3	0.04246
1	2933	2450096.9	0.00737	-0.5	0.01961
1	2946	2462075.4	0.01531	0.2	0.02547
1	2983	2462038.3	0.00924	-0.7	0.08923
1	3097	2458575.5	0.01177	-1.0	0.17691
1	3315	2457673.7	0.00805	-0.9	0.08525
1	3377	2462946.6	0.04714	0.4	0.08766
1	3597	2452867.1	0.02380	-0.2	0.19275
1	3695	2458549.9	0.03182	1.0	0.07252
1	3741	2463602.8	0.00501	0.9	0.06418
1	3823	2464510.7	0.04062	0.2	0.46129
1	3834	2464326.7	0.00763	1.0	0.36717
1	3857	2456296.5	0.01847	0.8	0.02895
1	3963	2463688.7	0.01153	-0.3	0.03518
1	4046	2464555.9	0.01012	0.8	0.03584
1	4176	2454528.7	0.04948	0.7	0.13132
1	4195	2462938.7	0.04233	-0.5	0.05481
1	4265	2465299.3	0.02247	0.2	0.02689
1	4292	2457997.0	0.02752	-0.4	0.07412
1	4292	2463811.0	0.01424	-1.0	0.12347
1	4325	2454662.0	0.00550	0.9	0.06033
1	4428	2466312.3	0.01042	-0.4	0.05577
1	4434	2465274.9	0.02145	0.7	0.02509
1	4467	2460684.0	0.02406	-0.6	0.03376
1	4474	2465370.8	0.00379	-0.5	1.21987
1	4513	2451093.5	0.00566	0.8	0.05319

TABLE II
Final Results for 208 Pairs

Ast.1	Ast.2	D_1	D_2	JD	y-m-d	ρ	V_r	ΔV_2	r_1
1	1393	913		2459718.5	2022 05 19	0.006908	2.207	68662	2.617
3	3173	244		2451981.0	2001 03 12	0.008749	9.399	243	2.054
3	3563	244		2464653.2	2035 11 21	0.007924	5.665	445	2.211
4	2831	501		2461792.2	2028 01 21	0.007028	4.502	5467	2.477
5	2912	125		2454162.0	2007 03 02	0.006387	2.853	147	2.364
5	4404	125		2449887.7	1995 06 19	0.008088	11.303	29	2.087
6	524	192	74	2458661.3	2019 06 26	0.004334	7.205	312	2.616
6	4497	192		2449202.1	1993 08 02	0.004111	3.450	686	2.883
7	2346	203		2450442.1	1996 12 24	0.007772	3.751	395	2.727
7	4254	203		2463877.1	2033 10 06	0.002829	3.514	1157	2.674
8	330	141		2449550.8	1994 07 17	0.008288	1.696	274	1.872
8	836	141		2453265.1	2004 09 16	0.009291	3.187	130	1.902
8	967	141	15	2450697.0	1997 09 05	0.004307	4.435	202	1.919
8	2200	141		2458220.9	2018 04 12	0.004777	6.620	122	2.197
8	3137	141		2450983.5	1998 06 19	0.009749	3.279	121	2.054
8	3393	141		2463223.8	2031 12 23	0.005768	3.597	186	2.510
10	2061	429		2451272.5	1999 04 04	0.006463	9.418	1783	3.046
11	17	162	93	2450452.2	1997 01 03	0.005362	2.358	463	2.212
11	2204	162	28	2454795.2	2008 11 24	0.003418	10.968	156	2.230
11	3969	162		2461587.3	2027 06 30	0.005732	1.960	521	2.261
12	1110	117		2456600.0	2013 11 03	0.004638	1.224	388	2.174
12	2365	117		2465632.1	2038 07 27	0.006721	6.615	50	2.395
12	3658	117		2455253.1	2010 02 25	0.008220	6.014	45	2.291
12	4319	117		2458269.6	2018 05 31	0.007385	4.712	63	2.021
13	1120	215		2461162.2	2026 05 01	0.008193	6.416	260	2.547
13	3490	215		2452238.5	2001 11 25	0.009355	4.481	326	2.661
15	765	272		2455351.4	2010 06 03	0.003447	2.726	2946	2.824
15	3137	272		2458604.8	2019 05 01	0.009583	3.069	941	2.621
16	70	264	127	2466318.1	2040 06 12	0.005988	5.261	803	2.523
16	3965	264		2465904.8	2039 04 26	0.004286	4.628	1276	2.832
20	3402	151		2450192.4	1996 04 18	0.009918	8.305	57	2.291
22	2222	187	28	2457598.7	2016 07 29	0.008298	4.009	270	2.613
22	4181	187		2459234.2	2021 01 19	0.002552	6.196	569	2.702
23	1088	111		2453957.5	2006 08 10	0.004472	6.520	65	2.066
23	3410	111		2460168.3	2023 08 11	0.007107	4.459	59	2.093
23	3529	111		2454395.0	2007 10 21	0.007908	7.377	32	2.693
23	4214	111		2455486.3	2010 10 16	0.006372	4.114	72	2.112
28	1629	126	11	2465508.4	2038 03 25	0.002076	3.435	386	2.584
28	4056	126		2452275.6	2002 01 01	0.005199	1.499	353	2.369
29	37	219	112	2465278.8	2037 08 06	0.006032	2.164	107	2.593
29	987	219	45	2449414.6	1994 03 03	0.002438	3.198	853	2.688
29	4342	219		2457103.4	2015 03 21	0.009600	4.266	353	2.522
30	1493	104	26	2462446.1	2029 11 05	0.004064	3.500	109	2.068
30	2593	104		2459538.3	2021 11 19	0.007495	3.059	67	2.334
30	2663	104		2453208.6	2004 07 22	0.007616	3.891	52	2.070
30	2712	104		2460931.6	2025 09 13	0.006054	2.681	95	2.240
30	3888	104		2461025.4	2025 12 15	0.005668	10.896	25	2.134
30	3961	104		2450284.6	1996 07 20	0.008999	5.587	31	2.261
34	1736	118	30	2459942.8	2022 12 29	0.009241	2.932	83	2.603
34	2580	118		2463026.3	2031 06 08	0.008849	4.816	53	2.473
35	2675	108		2449786.3	1995 03 09	0.006875	4.422	57	2.424
36	4023	109		2452118.8	2001 07 28	0.009151	8.383	23	2.029
37	243	112	33	2454509.7	2008 02 13	0.009966	2.011	96	2.952
37	2054	112	25	2462585.7	2030 03 25	0.008925	2.011	108	3.113
37	4290	112		2462480.4	2029 12 09	0.007415	4.186	62	3.062
38	2558	120		2453545.8	2005 06 24	0.005070	5.372	87	2.340
39	920	159	27	2465139.4	2037 03 21	0.004334	3.467	368	2.854

Table II—*Continued*

Ast.1	Ast.2	D_1	D_2	JD	y-m-d	ρ	V_r	ΔV_2	r_1
39	2601	159	23	2451005.5	1998 07 11	0.009204	5.761	104	2.938
40	813	111	16	2458247.8	2018 05 09	0.008161	1.552	149	2.162
40	1680	111	16	2460118.3	2023 06 22	0.005835	2.971	109	2.373
40	1830	111		2459573.8	2021 12 25	0.002994	2.100	299	2.177
42	1367	107		2459074.5	2020 08 13	0.002591	12.471	52	2.040
42	2139	107		2459582.4	2022 01 02	0.009525	3.302	54	2.612
45	1406	214	31	2458084.3	2017 11 26	0.009056	6.872	217	2.685
45	3481	214		2465124.6	2037 03 07	0.008496	3.121	508	2.528
46	3102	131		2466323.6	2040 06 18	0.003956	5.228	150	2.952
51	265	153	31	2454848.4	2009 01 16	0.006753	11.717	62	2.515
52	1716	312	29	2465184.6	2037 05 06	0.008725	3.972	1205	2.756
52	1949	312		2465236.0	2037 06 26	0.007112	5.141	1143	2.747
53	467	119	48	2451640.0	2000 04 05	0.009567	3.687	66	2.917
53	1088	119		2455574.5	2011 01 13	0.007936	5.249	56	2.629
53	2284	119		2456524.2	2013 08 19	0.006331	4.181	88	2.438
56	4653	117		2455828.5	2011 09 24	0.008275	2.970	90	2.259
59	3848	173		2451203.6	1999 01 25	0.005187	3.739	367	2.450
63	839	108	22	2459627.0	2022 02 16	0.004509	2.884	133	2.235
63	3813	108		2460528.5	2024 08 06	0.002340	1.435	516	2.312
68	3669	127		2452341.2	2002 03 07	0.008663	3.716	88	2.286
69	671	143	64	2465935.3	2039 05 26	0.007846	5.135	100	3.092
70	3373	127		2451840.0	2000 10 22	0.004949	6.618	86	2.491
74	2351	123		2460650.4	2024 12 05	0.009628	3.037	88	2.138
74	3505	123		2464588.0	2035 09 17	0.001522	3.526	477	3.114
74	3562	123		2462543.6	2030 02 11	0.009573	7.238	37	2.287
74	3762	123		2455681.9	2011 04 30	0.008960	2.033	141	2.135
78	1272	125		2449856.3	1995 05 18	0.008416	6.392	50	2.711
78	2227	125		2465086.2	2037 01 27	0.004503	5.284	113	2.162
81	473	124		2449426.7	1994 03 15	0.007075	4.242	87	2.949
81	3798	124		2462063.7	2028 10 19	0.007876	4.027	83	2.335
85	4629	157		2462560.4	2030 02 27	0.008138	8.422	78	2.334
89	559	159	80	2452533.1	2002 09 15	0.006365	8.418	103	2.783
89	1541	159	22	2451002.0	1998 07 07	0.005772	5.976	160	2.709
91	3868	114		2461270.6	2026 08 18	0.004237	3.836	125	2.565
98	1622	109		2459691.3	2022 04 21	0.008271	5.729	38	2.196
107	2231	237		2463864.1	2033 09 23	0.008018	5.418	421	3.347
107	4406	237		2448626.4	1992 01 04	0.009774	3.101	604	3.536
111	2455	139		2452965.4	2003 11 21	0.006009	1.615	381	2.857
111	3773	139		2465956.2	2039 06 16	0.001432	4.234	609	2.329
114	2433	103		2453008.9	2004 01 04	0.002815	5.499	97	2.881
114	3388	103		2466752.8	2041 08 21	0.005051	6.949	43	2.752
120	1637	178	50	2449321.2	1993 11 29	0.009202	3.128	270	2.918
120	1755	178	29	2453930.7	2006 07 14	0.000872	5.160	1724	3.216
128	2652	194		2464878.5	2036 07 04	0.006479	0.988	1569	2.419
128	3971	194		2462846.9	2030 12 11	0.008198	5.846	210	2.787
130	3556	189		2449257.1	1993 09 26	0.008384	7.226	153	3.126
134	376	122	37	2451945.7	2001 02 05	0.003800	5.929	111	2.383
137	3321	150		2454057.2	2006 11 17	0.009235	3.357	150	2.827
139	1479	162		2465760.8	2038 12 03	0.005401	4.136	262	2.314
139	3037	162	21	2448730.2	1992 04 17	0.009294	9.894	64	2.350
140	2160	114		2450446.6	1996 12 29	0.007513	2.471	110	2.767
140	2473	114		2451458.4	1999 10 06	0.009665	5.163	46	2.529
140	2739	114	14	2464867.5	2036 06 23	0.003989	1.900	269	2.161
141	3539	135		2459934.0	2022 12 20	0.003309	9.184	111	2.884
144	1218	146		2458303.5	2018 07 04	0.009508	5.601	80	2.473
144	2751	146	20	2466070.8	2039 10 09	0.004846	5.089	174	2.802
144	4253	146		2464768.2	2036 03 15	0.008425	6.600	77	2.160
145	3688	155		2449188.9	1993 07 20	0.007491	7.635	90	2.976
146	2639	137		2454477.4	2008 01 11	0.006179	1.836	312	2.823
146	3059	137		2464976.6	2036 10 10	0.008898	4.765	83	2.552
148	3152	104	35	2458301.2	2018 07 01	0.005114	12.572	24	2.593
150	1097	157	25	2457186.3	2015 06 12	0.009214	3.366	172	2.822
150	1097	157	25	2466591.1	2041 03 12	0.009399	3.410	166	2.802

Table II—*Continued*

Ast.1	Ast.2	D_1	D_2	JD	y-m-d	ρ	V_r	ΔV_2	r_1
150	1725	157		2459140.0	2020 10 17	0.004349	1.316	930	2.906
150	2231	157		2454009.7	2006 10 01	0.007364	2.888	250	3.364
185	953	165	32	2463382.5	2032 05 30	0.009241	8.702	77	3.046
191	1605	105	39	2454470.5	2008 01 05	0.008381	1.675	113	2.775
191	2613	105		2456414.4	2013 05 01	0.009782	6.591	25	2.910
191	4352	105		2456851.4	2014 07 12	0.007679	4.848	43	3.164
192	2446	107		2458971.8	2020 05 02	0.005969	6.455	44	2.601
192	4651	107		2457766.3	2017 01 12	0.006440	3.314	79	2.887
194	779	174	73	2450404.5	1996 11 17	0.006179	7.394	159	3.225
200	2621	132	50	2455422.3	2010 08 13	0.002314	5.687	240	2.872
203	908	120	28	2456824.4	2014 06 15	0.000503	4.502	1050	2.828
203	1245	120	28	2448839.7	1992 08 05	0.007890	2.075	145	2.659
203	3453	120		2455859.5	2011 10 25	0.005879	1.726	234	2.596
212	1828	140	31	2452020.2	2001 04 20	0.003438	4.800	229	3.257
212	3216	140		2451723.7	2000 06 28	0.009149	4.031	102	2.939
216	2012	140		2463353.4	2032 04 30	0.007638	6.518	76	2.119
216	4229	140		2463454.5	2032 08 10	0.007538	6.653	75	2.141
221	724	110		2451174.2	1998 12 26	0.003161	4.057	143	2.927
230	1632	113	31	2452778.6	2003 05 19	0.009785	2.459	82	2.399
230	3172	113		2449977.0	1995 09 16	0.001794	5.277	210	2.476
230	4201	113		2450042.5	1995 11 21	0.005546	2.892	124	2.439
233	4226	108		2460461.9	2024 05 31	0.006775	2.962	86	2.580
240	77	108	71	2449713.9	1994 12 27	0.008923	1.853	105	2.628
241	1644	169		2461575.4	2027 06 18	0.008713	3.052	250	2.930
266	4399	113		2450430.3	1996 12 12	0.009763	4.100	50	2.383
308	231	148	85	2451734.1	2000 07 08	0.006602	3.628	186	2.792
308	1340	148	30	2455090.9	2009 09 16	0.005821	1.959	391	2.790
308	2166	148		2463084.1	2031 08 05	0.004710	1.828	518	2.852
313	269	101	55	2461595.1	2027 07 08	0.007373	7.451	26	2.536
313	442	101	68	2459502.1	2021 10 14	0.007872	4.840	37	2.184
313	3669	101		2466172.4	2040 01 18	0.007899	4.839	37	2.160
326	847	100	32	2456603.2	2013 11 06	0.004628	8.893	33	2.524
326	1652	100		2466393.3	2040 08 26	0.006455	10.075	21	2.003
344	1622	138		2462348.7	2029 07 31	0.006104	6.678	89	1.879
344	3344	138		2456580.8	2013 10 15	0.006753	5.329	100	2.432
345	101	100	68	2462789.4	2030 10 14	0.008799	6.981	22	2.279
345	413	100	34	2451377.1	1999 07 17	0.008975	10.859	14	2.184
345	1242	100	49	2458916.3	2020 03 07	0.007225	6.786	28	2.274
345	1791	100	29	2449297.6	1993 11 06	0.008172	3.296	51	2.440
345	4039	100		2449817.5	1995 04 10	0.003903	3.066	115	2.295
346	3143	110		2453585.5	2005 08 03	0.009867	1.964	94	2.689
346	3301	110		2460244.1	2023 10 26	0.008355	2.983	73	2.550
346	4136	110		2450051.7	1995 11 30	0.006110	4.181	72	2.585
349	1400	143		2449724.8	1995 01 07	0.002843	7.552	187	3.174
349	2963	143		2465207.1	2037 05 28	0.004423	1.936	470	2.670
354	684	162		2455297.6	2010 04 11	0.003847	8.070	188	2.475
354	2752	162		2455933.4	2012 01 06	0.008935	5.020	130	3.028
356	3302	135		2451096.0	1998 10 09	0.009861	6.242	55	2.661
356	3350	135		2464486.8	2035 06 08	0.008644	6.536	60	2.701
360	3131	121		2452296.2	2002 01 21	0.009689	5.197	48	2.846
372	1392	195	30	2454129.6	2007 01 29	0.008302	5.453	225	2.425
386	481	173	116	2451819.3	2000 10 01	0.005201	8.586	159	2.408
387	1904	106	21	2457701.6	2016 11 09	0.004282	4.862	78	2.928
393	336	106	72	2451913.6	2001 01 04	0.006183	5.680	47	2.329
404	2870	101		2451509.0	1999 11 26	0.007223	8.324	24	2.278
405	2336	129		2449055.5	1993 03 09	0.009272	5.460	58	2.849
405	4570	129		2449552.8	1994 07 19	0.009062	5.495	59	1.988
409	1817	168	17	2452855.5	2003 08 04	0.006817	10.627	90	2.733
409	3383	168		2465055.4	2036 12 27	0.009662	6.331	107	2.687
409	3999	168		2454977.4	2009 05 25	0.006556	4.200	237	2.395
410	3530	128		2458683.5	2019 07 19	0.007568	6.566	58	2.067
426	693	134	69	2454962.6	2009 05 11	0.007655	4.340	100	2.874
444	597	170	38	2461035.0	2025 12 25	0.009579	8.077	87	2.370

Table II—*Continued*

Ast.1	Ast.2	D_1	D_2	JD	y-m-d	ρ	V_r	ΔV_2	r_1
469	4168	129		2448834.1	1992 07 30	0.003701	1.987	402	3.001
476	2141	121	27	2461178.4	2026 05 17	0.008942	2.333	117	2.465
481	299	116	21	2452009.8	2001 04 10	0.008030	4.227	63	2.305
490	1502	121	35	2455205.3	2010 01 08	0.008389	2.195	132	2.907
506	124	109	80	2451003.0	1998 07 08	0.002210	6.864	117	2.684
521	4077	121		2463333.2	2032 04 10	0.006289	4.920	79	3.192
602	966	130	27	2462597.7	2030 04 06	0.009679	8.878	35	2.960
618	3820	124		2452436.5	2002 06 11	0.005022	7.896	66	2.985
626	1928	104		2456936.6	2014 10 06	0.008846	8.666	20	2.975
626	4351	104		2455408.5	2010 07 31	0.004353	8.293	43	2.936
654	1235	132		2455107.9	2009 10 03	0.005369	11.413	52	2.202
654	2352	132		2454348.6	2007 09 05	0.001853	5.048	338	2.780
654	2430	132		2460273.2	2023 11 24	0.009611	14.497	23	2.394
683	3474	116		2459620.0	2022 02 09	0.006900	5.854	53	3.081
704	651	333	37	2457499.1	2016 04 20	0.005417	6.789	1381	3.220
704	977	333	67	2453653.8	2005 10 10	0.009135	9.519	584	3.144
704	1484	333	46	2458351.4	2018 08 20	0.006956	11.823	618	2.846
705	3093	139		2460001.4	2023 02 25	0.007704	8.613	56	2.959
751	1166	115	24	2453853.5	2006 04 28	0.007509	5.396	52	2.824
769	1192	102		2451119.5	1998 11 02	0.006381	8.273	28	2.794
772	663	123	104	2449264.6	1993 10 04	0.005810	14.830	30	2.743
814	1377	116		2457761.6	2017 01 08	0.009549	10.840	21	2.412
1467	494	112	89	2449548.8	1994 07 15	0.009097	6.772	31	2.887
1093	3085	120		2452740.9	2003 04 11	0.007499	10.343	31	2.623
1902	4120	101		2456855.1	2014 07 16	0.008704	5.283	31	3.436

Note. Diameters are taken from asteroid data base kindly provided by E. Tedesco. V_r is the relative velocity, ΔV is the change of the velocity in the perturbed asteroid resulting from the mutual close approach, and r_1 is the heliocentric distance of a perturbing asteroid at the epoch of minimum distance. D_1 and D_2 are given in km, ρ and r_1 in AU, V_r in km/sec, and ΔV_2 in km/sec × 10^{-9}.

examine, resulting at the end in a reasonable number of pairs which then undergo a detailed analysis—numerical integration involving all the perturbing planets (but in the time span of few days centered on the approximate epoch of the minimum distance), mutual perturbations computation, estimation of the expected orbital changes, etc.

Computation of the minimum distances between the orbits of the two asteroids is a matter of applying the well-known spherical trigonometry procedures. Since method is described in detail in Lazović (1967) and Lazović and Kuzmanoski (1978), we are not going to quote here the equations; instead, let us just state that this computation involves the determination of the true anomalies of the relative nodes which serve as initial values and an iterative derivation of the corresponding true anomalies of the points where the orbits are closest to each other. As mentioned before, we have used only pairs with a minimum distance between orbits of the two bodies of less than 0.05 AU.

In order to find which pair of asteroids on nearby orbits can have a close encounter in a given time span, we apply the simple two-body approximation and first compute the mean anomaly of the point corresponding to the orbital proximity on the orbit of one of the asteroids and the associated initial epoch (1991). Then, we derive the mean anomaly of the other body in the pair corresponding to the same initial instant and to the subsequent instances obtained by subsequent adding of the time interval equal to the first body's revolution period up to the year 2041. Finally, we look at a given time span for all the occurrences of the other body in the vicinity of the proximity point (within a range of ±1° of mean anomaly) on its orbit. Since using the osculating mean motions for a particular epoch in the time spans comparable or longer than the typical periods of the short-periodic perturbations very quickly accumulates the error in the mean anomaly, we employed in this step two complementary sets of initial elements: the osculating elements of 4722 asteroids for the epoch December 10, 1991 (kindly provided by B. Marsden), and a set of their mean elements derived by an analytical method (Knežević, 1988). Thus, after completing the above two steps of our selection procedure, for a total of 241 asteroids larger than 100 km in diameter (as inferred from the asteroid data base by E. Tedesco), and out of more than a million possible pairs, we found some 20,000 pairs satisfying our criteria and entering the next phase (note that some of these were in fact doubles—recognized by using both the osculating elements and the mean elements as initial data sets).

Clearly, the previous steps had provided only the unperturbed data, so next we had to include the effects of perturbations. Therefore, we performed a numerical inte-

TABLE III

Maximum Expected Differences of the Positions of the Perturbed Asteroids (Ast. 2), Due to Effects by the Perturbing Asteroids (Ast. 1)

Ast.1	Ast.2	M_1	JD	y-m-d	$\Delta\alpha$	$\Delta\delta$	Δ
1	1393	59.00	2460396	2024 03 26	$-0^{s}.2989$	$-0''.564$	$4''.069$
4	2831	12.00	2462672	2030 06 19	0.0850	0.603	1.407
6	4497	0.55	2450202	1996 04 28	0.0063	0.021	0.091
7	4254	0.64	2464877	2036 07 02	-0.0074	-0.016	0.111
10	2061	6.12	2452162	2001 09 09	0.0519	0.269	0.817
15	765	1.56	2455921	2011 12 25	0.0154	-0.046	0.213
16	70	1.43	2467318	2043 03 09	-0.0096	0.075	0.157
16	3965	1.43	2466905	2042 01 20	0.0060	-0.039	0.095
29	37	0.81	2466278	2040 05 03	0.0101	0.073	0.168
29	987	0.81	2450415	1996 11 27	0.0053	-0.061	0.132
52	1716	2.35	2466185	2040 01 31	-0.0066	-0.039	0.106
52	1949	2.35	2466206	2040 02 21	0.0133	0.074	0.212
107	4406	1.03	2449626	1994 09 30	0.0032	-0.021	0.052
111	3773	0.21	2466818	2041 10 25	-0.0078	-0.049	0.127
120	1755	0.44	2454931	2009 04 09	0.0150	-0.049	0.229
128	2652	0.57	2465879	2039 03 31	0.0052	-0.042	0.086
150	1725	0.30	2460140	2023 07 14	0.0088	-0.013	0.124
203	908	0.13	2457824	2017 03 11	-0.0040	0.034	0.068
704	651	2.86	2458497	2019 01 13	-0.0085	0.003	0.108

Note. M_1 is the approximate mass of a perturbing asteroid in units of $10^{-11}\ M_{\odot}$ and $\Delta = \sqrt{(\Delta\alpha \cos\delta)^2 + (\Delta\delta)^2}$. Only cases with $\Delta > 0''.05$ are listed.

gration for all the pairs, covering in each case the interval from the common osculating epoch of the orbital elements to an epoch 10 days before the possible close encounter; then we computed for both asteroids in a pair a two-body ephemerides for the next 20 days with a step of only 0.1 day. Here we employed the software ORBIT8V (kindly provided by A. Milani), which is based on a simplified dynamic model including the four outer major planets and a barycentric correction accounting for the effect of the inner planets. This integration is efficient enough and it furnishes the reliable data for the final selection of potentially important close encounters. Eventually, we found 208 pairs of asteroids which should in the next 50 years reach mutual distances less than 0.01 AU.

Let us emphasize here that these 208 close approaches to within 0.01 AU that we found are not all the possible such encounters among the selected asteroids occurring in the investigated time span. The weak points of our procedure are the use of the two-body propagation in the second phase of our selection procedure and the rather restrictive criteria introduced in order to reduce the amount of necessary computations in the third phase of selection. So, for example, almost two-thirds of the candidate pairs did not meet even the 0.05 AU distance criterion after the application of the perturbations; on the other hand, this loss is not compensated by the pairs which start to fulfill the conditions only after perturbations are applied, because these were previously eliminated. Hence, the real number of close encounters should be somewhat larger (300–400 as predicted by Farinella and Davis 1992, and obtained by Yoshikawa and Nakamura, 1992) than that we found here. To find them all would be possible (by using several sets of osculating elements for different epochs, for example), but this was beyond the scope of this paper; rather, as already pointed out, we were interested in finding whether we can find some close encounters giving rise to measurable perturbations, which are the most interesting cases, and how large the expected effects are.

For example, Table I summarizes the results of our selection procedure and illustrates all the above-described steps; it shows only the close encounters with 1 Ceres we found by using the osculating elements as the initial data set, giving for each pair the approximate epoch of the minimum distance, the minimum distance between the osculating orbits ρ_{orb}, the difference ΔM of the mean anomalies of the second body and the proximity point on its orbit, and the approximate mutual distance ρ (corresponding to the approximate epoch of the minimum distance) of the bodies obtained by numerical integration in the framework of the simplified Solar System model. Obviously, only one of the represented pairs (1, 1393) fulfills the adopted distance criterion for entering the list of the selected 208 close encounters which were to be more closely examined.

3. RESULTS AND DISCUSSION

In deriving the exact values for the epochs, distances, relative velocities, and radius vectors for 208 selected close encounters we made use of a Radau integrator of

TABLE IV
The Expected Differences of Positions of Asteroid 1393 Sofala Due to Perturbations by 1 Ceres in Two Oppositions (JD 2460031 and JD 2460579) after the Close Encounter

JD	y-m-d	α	δ	$\Delta\alpha$	$\Delta\delta$
2459931.2	2022 12 17	$10^{h}39^{m}18^{s}.236$	$14^{\circ}43'32''.22$	$-0^{s}.040$	$-0''.01$
2459941.2	2022 12 27	10 49 36.585	13 39 00.74	-0.044	0.01
2459951.2	2023 01 06	10 59 55.492	12 32 13.49	-0.047	0.04
2459961.2	2023 01 16	11 10 15.242	11 23 16.31	-0.052	0.07
2459971.2	2023 01 26	11 20 36.157	10 12 15.63	-0.056	0.11
2459981.2	2023 02 05	11 30 58.596	8 59 18.36	-0.060	0.14
2459991.2	2023 02 15	11 41 22.953	7 44 31.96	-0.065	0.18
2460001.2	2023 02 25	11 51 49.654	6 28 04.52	-0.070	0.22
2460011.2	2023 03 07	12 02 19.157	5 10 04.68	-0.074	0.27
2460021.2	2023 03 17	12 12 51.947	3 50 41.77	-0.080	0.31
2460031.2	2023 03 27	12 23 28.539	2 30 05.76	-0.085	0.35
2460041.2	2023 04 06	12 34 09.469	1 08 27.31	-0.090	0.40
2460051.2	2023 04 16	12 44 55.293	-0 14 02.23	-0.096	0.45
2460061.2	2023 04 26	12 55 46.583	-1 37 10.75	-0.102	0.50
2460071.2	2023 05 06	13 06 43.923	-3 00 45.42	-0.108	0.54
2460081.2	2023 05 16	13 17 47.903	-4 24 32.68	-0.115	0.59
2460091.2	2023 05 26	13 28 59.113	-5 48 18.18	-0.122	0.63
2460101.2	2023 06 05	13 40 18.137	-7 11 46.89	-0.128	0.68
2460111.2	2023 06 15	13 51 45.541	-8 34 43.03	-0.136	0.72
2460121.2	2023 06 25	14 03 21.870	-9 56 50.16	-0.143	0.75
2460481.2	2024 06 19	21 50 25.310	-19 16 21.04	-0.266	-1.11
2460491.2	2024 06 29	22 01 07.884	-18 21 32.50	-0.261	-1.14
2460501.2	2024 07 09	22 11 37.171	-17 25 05.98	-0.256	-1.18
2460511.2	2024 07 19	22 21 53.655	-16 27 13.28	-0.252	-1.20
2460521.2	2024 07 29	22 31 57.860	-15 28 05.54	-0.247	-1.23
2460531.2	2024 08 08	22 41 50.339	-14 27 53.17	-0.242	-1.24
2460541.2	2024 08 18	22 51 31.667	-13 26 45.95	-0.237	-1.26
2460551.2	2024 08 28	23 01 02.430	-12 24 52.95	-0.233	-1.27
2460561.2	2024 09 07	23 10 23.225	-11 22 22.67	-0.228	-1.27
2460571.2	2024 09 17	23 19 34.647	-10 19 22.95	-0.224	-1.27
2460581.2	2024 09 27	23 28 37.294	-9 16 01.12	-0.219	-1.27
2460591.2	2024 10 07	23 37 31.754	-8 12 23.93	-0.215	-1.26
2460601.2	2024 10 17	23 46 18.610	-7 08 37.67	-0.211	-1.25
2460611.2	2024 10 27	23 54 58.436	-6 04 48.14	-0.207	-1.24
2460621.2	2024 11 06	0 03 31.791	-5 01 00.75	-0.203	-1.23
2460631.2	2024 11 16	0 11 59.223	-3 57 20.52	-0.199	-1.21
2460641.2	2024 11 26	0 20 21.269	-2 53 52.10	-0.196	-1.19
2460651.2	2024 12 06	0 28 38.449	-1 50 39.83	-0.192	-1.17
2460661.2	2024 12 16	0 36 51.271	-0 47 47.78	-0.188	-1.15
2460671.2	2024 12 26	0 45 00.229	0 14 40.25	-0.185	-1.13

order 15 developed by Everhart (1985) and adjusted by M. Carpino. Our integration by means of that program included perturbations by all the planets and covered periods from the common epoch of oculation of the initial asteroid orbits to the approximate epoch of the close encounter for each particular pair; then, starting from that approximate epoch, an integration with a very small step size has been made for a period of 10 days around it (results are given in Table II). Note that the differences between approximate and exact values of epochs and distances are small, but not negligible, especially from the point of view of asteroid mutual perturbations.

Clearly, the most interesting pairs are those for which mutual distance ρ and relative velocity V_r are small, and/or the first body in a pair is large. Hence, in the penultimate column of Table II we give the value $\Delta V_2 = 2GM_1/(\rho V_r)$, which represents the velocity change in the perturbed body resulting from the mutual close approach and combines all the three mentioned parameters. As suggested by Yeomans and Bender (private communication), this quantity can also be used as a parameter for the selection of the interesting pairs. It can be seen from Table II that pairs (1, 1393) involving the largest asteroid 1 Ceres, or (203, 908) due to the smallest close approach distance found so far, appear as the most promising ones for further investigation. Adding to these two a number of other pairs best suiting the aforementioned criteria ((4, 2831), (6, 4497), etc.), the mutual perturbations have been estimated for a period of 1300 days (starting 300 days before the close encounter and ending 1000 days after the close encounter). Masses of the perturbing asteroids have been estimated adopting for all of them the same density, equal to that of 1 Ceres, and adopting for the mass of Ceres a value of $5.9 \times 10^{-10} M_\odot$, recommended by the IAU (note that this was an entirely satisfactory approximation for our purpose). Since causing only completely negligible effects, Pluto has been discarded in all our subsequent calculations. We performed two integrations, once without the perturbing asteroid and the other time slotting it instead of Pluto in the integration scheme. The differences among the obtained positions of the perturbed asteroid have been assigned entirely to the effect of the perturbing one, thus providing directly an estimate of the expected effect of a close encounter.

The obtained results are given in Table III. As can be seen, only in the case of pair (1, 1393) are the expected differences really substantial, and effects should be easily measurable; this encounter, however, occurs only ~30 years from now and the potential outcome can have only a limited significance since the mass of 1 Ceres is already rather well known (Goffin 1991, Williams 1992). The next best case (4, 2831) is also very distant in time. Pair (10, 2061) is therefore perhaps the most interesting one (although there is also an estimate of the mass of 10 Hygiea—see Scholl *et al.* 1987); it is supposed to occur only some 8 years from now, and the expected effect is probably large enough to be accurately determined, if necessary, in the framework of a dedicated observational campaign (Yeomans, 1992, thus reports that such a campaign in the case of 951 Gaspra furnished positions with an accuracy of 0.″3). In the case of pair (203, 908), despite of the very close approach of the two asteroids, the expected effects are relatively small, due in the first place to the fact that perturbing asteroid 203 Pompeja is also a relatively small body. For all other pairs the mutual perturbing effects are rather small as well, but a reasonable improvement of the observational accuracy, the improvement of the accuracy of stellar catalogues (such as one expected as a result of the Hipparcos mission), organization of dedicated observational campaigns, etc., could help in taking full advantage of these data and provide a set of reliable asteroid masses.

Since encounters with 1 Ceres obviously deserve special attention, we decided to present here an additional result just for the pair (1, 1393). The expected differences of positions of the perturbed asteroid 1393 Sofala around a couple of oppositions following the close encounter are determined, and the results are shown in Table IV. Table IV gives the epochs, right ascensions and declinations of the perturbed asteroid, and differences in right ascensions and declinations obtained from the two integrations (with and without 1 Ceres). Although the accuracy of the computed ephemerides cannot be very high, it is clear that the expected differences in both covered oppositions are more than large enough to be precisely measured; thus, the observations which will be eventually collected on these occasions should enable an accurate determination of the mass of 1 Ceres. As we mentioned before, this result in itself has no particular significance, but it shows that the encounters with 1 Ceres occurring at even greater distances than the one found here will be possible to use (see Table I). This particular case, being the most favorable one in the immediate future, can, therefore, be employed, for example, to check and adjust all the other determinations.

Let us briefly consider in conclusion the question of the reliability of our results. As already mentioned, the criteria adopted here for the selection of asteroids which will approach each other to a small distance in the next 50 years can prevent one from recognizing some potentially interesting cases, but ensure that marginal or useless close encounters do not enter our list. The method of selection and mutual perturbations estimation is simple, only the standard procedures are employed, and the results can be easily checked. The (in)accuracy of the osculating elements of initial orbits should not affect the obtained results in a significant way, since we were dealing with numbered objects only; in fact, the possible errors in initial conditions can slightly change our minimum distances, but knowing the size of the typical residuals of orbital fits for numbered asteroids these changes should be essentially negligible for our purpose.

In this paper we have demonstrated a method of revealing close encounters among asteroids, and we found some encounters involving the largest asteroids and enabling derivation of useful results even at the current level of observational accuracy. In the near future, however, we plan to check systematically the past encounters and existing observations, to detect close encounters with nonnumbered asteroids with good quality orbits, even to search for the very close encounters with small asteroids not taken into account by Yoshikawa and Nakamura which could prove useful in case improvements in the observational accuracy, reliability of the orbital fits, and ephemerides are achieved in the meantime.

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