# EARTH'S MODEL WITH VARIABLE CHANDLER'S FREQUENCY

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МОДЕЛЬ ЗЕМЛИ С ПЕРЕМЕННОЙ ЧАНДЛЕРОВОЙ ЧАСТОТОЙ

Подтверждена гипотеза о нелинейной зависимости Чандлеровой частоты от тотальной амплитуды полярного движения из анализа координат полюса МСШ. Предполагая что нелинейная зависимость Чандлеровой частоты вызвана динамическим океаном мы ввели новую возбуждающую функцию динамического океана в модели Земли Вондрака. Выведена реакция этой модели Земли с переменной и амплитудно зависимой Чандлеровой частотой на атмосферное возбуждение.

The hypothesis of a nonlinear dependence of Chandler's frequency on the total polar motion amplitude is confirmed by analysing the ILS coordinates of the pole. Assuming that this nonlinear dependence of Chandler's frequency is due to a nonequilibrium ocean, one introduces a new excitation function of the nonequilibrium ocean in Vondrák's (1987) model of the Earth. The response of this more complete Earth's model with a variable and amplitude-dependent Chandler's frequency to atmospheric excitation is derived, as well.

**Key words:** Chandler's frequency — polar motion — atmospheric excitation

## 1. Introduction

One of the most important events in the history of polar motion research is Chandler's discovery of 1892 (Chandler 1892). He found, by analysing a large number of observations, that variations in geographic latitude have two periodic components: one named after Chandler himself, whose period is 427 days, and another with an annual period. Newcomb (1872) was first to attempt an explanation of both the difference between the Eulerian and Chandler's periods and the annual component. In determining the Eulerian period one has assumed that the Earth is an absolute solid body which is not the case; if its elasticity is taken into account, then a satisfactory agreement between theory and observations is reached.

The variability of the Chandler's period has been controversial for almost an entire century. A number of authors, including Chandler himself, have assumed that the Chandler's period is either multi-component, or variable in time (Chandler 1892, Kimura 1918, Hattori 1949, Melchior 1957, Colombo and Shapiro 1978, Gaposchkin 1972, Sekiguchi 1972, 1976, Carter 1981, Dickman 1981, Pejovic 1983, Vondrák 1985, 1988). There are other authors claiming that the Chandler's nutation has a single period (Newcomb 1892, Pedersen and Rochester 1972, Ooe 1978, Okubo 1982).

A variety of results as well as theoretical considerations have been presented in two most important monographs devoted to the Earth's rotation: Munk and MacDonald (1960) and Lambeck (1980).

More recent and more detailed information about the atmosphere has been presented in papers where the influence of atmospheric effects on the polar motion is considered (e.g. Wilson and Haubrich 1976a, 1976b, Jochman 1976, 1981a, 1981b, Lambeck and Hopgood 1981, Wahr 1982, 1983, Eubanks et al. 1977, Brzezinski 1987, Sidorenkov 1982, Barnes et al. 1983, Hide 1984, Dutton and Fallon 1985, Eubanks et al. 1985, Vondrák 1987, 1989, Salstein 1987). Among the important papers is certainly that published by Barnes et al. (1983) where a solid basis for calculating effective angular momentum functions (EAMF) for the atmosphere based on global meteorological data was given.

Attempts of using the computed atmospheric EAMF for the purpose of numerical integration of polar motion, carried out, e.g., by Barnes et al. (1983) or Dutton and Fallon (1985), have been successful in fitting the observed

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polar motion only over very limited time intervals (less than or equal to one Chandler's period); for longer time intervals both curves diverge very rapidly. A more realistic Earth's model has been used by Vondrák (1989) who achieved a better fit between the integrated and observed polar motions over a significantly longer time interval (9 years). The model characterised by triaxial Earth's structure, its fluid core, visco-elastic mantle and equilibrium ocean was proposed by the same author (Vondrák 1987). In order to improve the fit, a modified version of Vondrák (1987) model is proposed in the present paper. The modification is that, instead of Vondrák's equilibrium ocean, a nonequilibrium ocean is introduced. The reason for this modification are Vondrák's (1985, 1988) results achieved by analysing the observed polar motion over a long time interval (1860–1985). Vondrák's basic result is that the Chandler's frequency is a nonlinear function of the polar motion amplitude. On the other hand, Carter (1981) suggested that a nonequilibrium ocean could be the cause of variability of Chandler's frequency. According to an earlier analysis of the present author (Pejović 1983) concerning the ILS pole coordinates (Yumi and Yokoyama 1980) there is no single Chandler's peak.

## 2. Chandler's Frequency

Prior to introducing a new excitation function of the nonequilibrium ocean in Vondrák's (1987) model of the Earth, the starting hypothesis concerning the nonlinear dependence of the Chandler's frequency on the total polar motion amplitude will be re-examined.

It has already been mentioned above that the variability of Chandler's period has been controversial for almost an entire century. A number of authors, including Chandler himself, have introduced hypotheses about the multiplicity or variability of the period. Okubo (1982) in his extended analysis devoted to Chandler's frequency problem attempted to solve the existing controversy. According to his conclusion Chandler's motion is not stationary and the mean value of the fluctuating Chandler's period is 435 days. It seems that what to Okubo was a randomly fluctuating Chandler's motion, to Carter (1981) and other authors was frequency modulated whereby Carter looked for the causes of this modulation in excitation functions suggesting the nonequilibrium ocean as the possible cause. The problem became more clear with the advent of Vondrák's (1985, 1988) results concerning a functional dependence of Chandler's frequency on the total polar motion amplitude. Namely, Vondrák analysed a long time interval (1860–1985) of inhomogeneous pole coordinate data. Besides confirming the results of other authors (e.g. Guinot 1972, 1982, Dickman 1981, Pejović 1985), Vondrák (1985, 1988) found, in addition to a rapid change in Chandler's phase for the interval 1920–1940, another change for 1870–1890 since his analysis included the interval before 1900. The rapid change of Chandler's phase between 1920 and 1940 cannot be explained by the available series of pole coordinates. However, it is not the only one as seen from the interval 1870–1890. These rapid phase changes coincide with the minima of the total

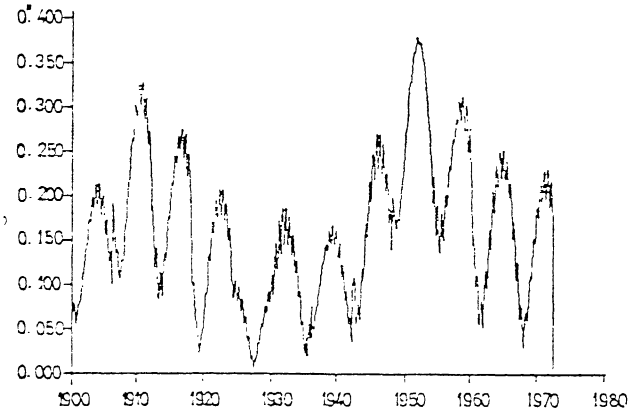


Fig. 1. The total amplitude of polar motion is obtained by analysing the ILS coordinates of the pole.

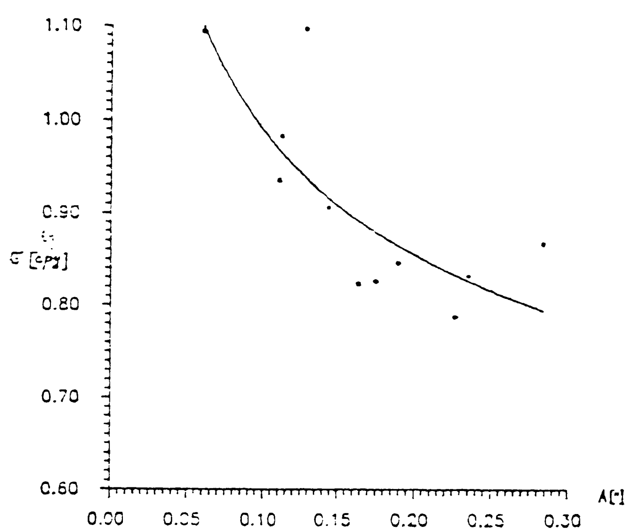


Fig. 2. Non-linear dependence between polar motion amplitude $A$ and Chandler frequency $\sigma$.

polar motion amplitude. The amount of correlation coefficient between Chandler's phase and the integral of the polar motion amplitude was found to be 0·986 (Vondrák 1985, 1988). Calculating the instantaneous Chandler's frequency from Chandler's phase differential, Vondrák also found Chandler's frequency to be an exponential function of the total polar motion amplitude (Vondrák 1988, Fig. 3).

In the present paper Vondrák's (1985, 1988) procedure has been re-applied to another homogeneous ILS pole-coordinate series and his results have been confirmed. The total polar motion amplitude displayed in Fig. 1 agrees well with Vondrák's (1985, Fig. 4) results. The instantaneous Chandler's frequency calculated from Chandler's phase differential is presented in Fig. 2 together with the total polar motion amplitude for every independent six-year interval. This figure confirms Vondrák's (1988, Fig. 3) results according to which Chandler's frequency is a nonlinear function of the polar motion amplitude.

## 3. Basic Equations

### 3.1. *Liouville Equations*

The Liouville equations can be written in a simple form used. e.g., by Munk and MacDonald (1960), Lambeck (1980), Barnes et al. (1983), Vondrák (1987)

$$m_1 - \frac{\dot{m}_2 A}{\Omega(C-A)} = \psi_1$$

$$m_2 + \frac{\dot{m}_1 A}{\Omega(C-A)} = \psi_2 \,, \tag{1}$$

where $\sigma_r = \Omega(C - A)/A$ is the Eulerian frequency, i.e. the free-nutation frequency of a rigid body, $C$ and $A$ are the polar and equatorial main inertia momenta of the nondeformed Earth, respectively, $\Omega$ is the angular velocity of the Earth's rotation, $m_1$ and $m_2$ are small quantities. In Eq. (1) $\psi_1$ and $\psi_2$ are the known excitation functions,

$$\psi_1 = \frac{\Omega^2\,\Delta I_{13} + \Omega\,\Delta\dot{I}_{23} + \Omega h_1 + \dot{h}_2 - L_2}{\Omega^2(C-A)}$$

$$\psi_2 = \frac{\Omega^2\,\Delta I_{23} - \Omega\,\Delta\dot{I}_{13} + \Omega h_2 - \dot{h}_1 + L_1}{\Omega^2(C-A)} \,, \tag{2}$$

where $(\dot{})$ denotes the time derivative in the rotating frame. Eq. (2) can be written in a conventional complex notation (Barnes et al. 1983)

$$\psi = \psi_1 + \mathrm{i}\psi_2 = \frac{\Omega^2\,\Delta I - \mathrm{i}\Omega\,\Delta\dot{I} + \Omega h - \mathrm{i}\dot{h} + iL}{\Omega^2(C-A)} \,,$$

where

$$\Delta I \equiv \Delta I_{13} + \mathrm{i}\,\Delta I_{23} \,,$$

$$h = h_1 + \mathrm{i}h_2 \,.$$

$$L = L_1 + \mathrm{i}L_2 \,.$$

By introducing $m = m_1 + \mathrm{i}m_2$ and $\dot{m} = \dot{m}_1 + \mathrm{i}\dot{m}_2$ equations (1) can be expressed in complex form

$$m + \mathrm{i}\dot{m}\frac{A}{\Omega(C-A)} = \psi \,. \tag{3}$$

It is known that this equation can be solved if the parts of the excitation function $\psi$ from the right-hand side of (3), which depend on $m$, are separated from the other parts and moved to the left-hand side of (3).

### 3.2. *Fluid Core*

The effects of a homogeneous fluid core on the rotation of a solid mantle are given in (Vondrák 1987). The parts of the excitation functions expressing the influence of the core will now be treated:

$$\Delta\psi_1^c = \frac{A_c\dot{V}_2 + A_c\Omega V_1}{\Omega^2(C-A)} \approx \frac{-A_c}{\Omega(C-A)}\dot{m}_2$$

$$\Delta\psi_2^c = \frac{A_c\Omega V_2 - A_c\dot{V}_1}{\Omega^2(C-A)} \approx \frac{A_c}{\Omega(C-A)}\dot{m}_1\,, \tag{4}$$

where $V_1$, $V_2$ are the components of vector of rotation of the core with respect to the mantle.

The equations of the system can be written in complex notation as

$$\Delta\psi^c = i\dot{m}\frac{A_c}{\Omega(C-A)}\,. \tag{5}$$

### 3.3. *Rotational Mantle Deformation*

The changes of the inertia tensor corresponding to the elastic rotational deformations are given in Vondrák (1984, 1987). The excitation functions corresponding to the elastic rotational deformation are

$$\Delta\psi_1^{rd} = \frac{\Delta I_{13}\Omega + \Delta\dot{I}_{23}}{\Omega(C-A)} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m_1 + \frac{\dot{m}_2}{\Omega}\right)$$

$$\Delta\psi_2^{rd} = \frac{\Delta I_{23}\Omega - \Delta\dot{I}_{13}}{\Omega(C-A)} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m_2 - \frac{\dot{m}_1}{\Omega}\right)$$

and, if one introduces the quality factor $Q$ for the viscosity as Lambeck (1980) did,

$$\begin{pmatrix}\tilde{m}_1\\ \tilde{m}_2\end{pmatrix} = \begin{pmatrix}\cos\varepsilon & \sin\varepsilon\\ -\sin\varepsilon & \cos\varepsilon\end{pmatrix}\begin{pmatrix}m_1\\ m_2\end{pmatrix}$$

where for small angles one has $\cos\varepsilon \approx 1$, $\sin\varepsilon \approx Q^{-1}$, one obtains the following expressions:

$$\tilde{m}_1 = m_1 + Q^{-1}m_2$$

$$\tilde{m}_2 = m_2 - Q^{-1}m_1\,.$$

If the latter expression is inserted into the former system of excitation-function equations, they will transform to

$$\Delta\psi_1^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left[m_1 + Q^{-1}m_2 + \frac{\dot{m}_2 - Q^{-1}\dot{m}_1}{\Omega}\right]$$

$$\Delta\psi_2^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left[m_2 - Q^{-1}m_1 - \frac{\dot{m}_1 + Q^{-1}\dot{m}_2}{\Omega}\right]. \tag{6}$$

The equations of the system can be written in complex notation as follows:

$$\Delta\psi^{rd} = \Delta\psi_1^{rd} + \mathrm{i}\,\Delta\psi_2^{rd} = \frac{k\Omega^2R^5}{3G(C-A)}\left(m - \mathrm{i}\frac{\dot{m}}{\Omega}\right)(1 - \mathrm{i}Q^{-1})$$

or, by introducing the secular Love number (Lambeck, 1980)

$$k_s = \frac{3G(C-A)}{r^5\Omega^2}\,,$$

the former equation becomes

$$\Delta\psi^{rd} = \frac{k}{k_s}\left(m - \mathrm{i}\frac{\dot{m}}{\Omega}\right)(1 - \mathrm{i}Q^{-1})\,. \tag{7}$$

### 3.4. *Nonequilibrium Ocean*

Because of Vondrák's (1988) results as well as the present ones concerning the nonlinear dependence of the Chandler frequency on the total amplitude of polar motion and Carter's (1981) suggestion according to which the cause of this dependence could be the nonequilibrium ocean, the excitation function of the nonequilibrium ocean in a complex notation is introduced in the following form:

$$\Delta\psi^0 = m\xi\,, \tag{8}$$

where $\xi$ is a factor depending on the polar motion amplitude. With Eq. (8) a new ocean function is introduced because, for the case of an equilibrium ocean, the excitation functions are given in Lambeck (1980):

$$\left.\begin{matrix}\Delta\psi_1\\ \Delta\psi_2\end{matrix}\right\} = \frac{3}{10}\frac{\varrho_\omega}{\varrho}\frac{1}{k_s}(1 + k - k')\left\{\begin{matrix}m_1A_1 + m_2A_2\\ m_1B_1 + m_2B_2\end{matrix}\right.$$

where $\varrho_\omega$ and $\varrho$ are the densities of the ocean and Earth, respectively, $k_s$, $k$ and $k'$ are the Love numbers; $m_1$ and $m_2$ are the coordinates of the pole along the zero meridian and $90E$, respectively; the constants $A_1$, $A_2$, $B_1$ and $B_2$ depend on the asymmetric ocean distribution over the Earth. Using the Lambeck formulae and the ocean function expansion (Lambeck, 1980) one finds: $A_1 = 1{\cdot}724$, $A_2 = B_1 = -0{\cdot}025$, $B_2 = 1{\cdot}406$. In this way one concludes that for the case of an equilibrium ocean symmetrically distributed upon the Earth the factor $\xi$ is approximately equal to 0·064. One should find the dependence of the $\xi$ factor on the polar motion amplitude $A$ for the case of a nonequilibrium ocean. Using the results of Vondrák (1988, Fig. 3), who found an exponential dependence between Chandler's frequency $\sigma$ and the polar motion amplitude $A$ by analysing the observed polar motion over a long time interval, (1860–1985), it is sufficient to apply

$$\xi = 0{\cdot}064(1 - e^{-20A})\,. \tag{9}$$

This yields

$$\Delta\psi^0 = 0{\cdot}064m(1 - e^{-20A}) \tag{10}$$

for the excitation function (8) in the case of the nonequilibrium ocean.

The excitation function (10) is very near the one for the equilibrium ocean:

$$\Delta\psi_s^0 = 0{\cdot}064m\,.$$

They differ only in the case of very small amplitudes and they are equal to each other in the case of amplitudes greater than 0·2″.

## 4. Solution of the Liouville Equation in the Case of Variable Chandler's Frequency

Substituting all the derived excitations (5), (7) and (8) in the left-hand side of the Liouville equation (3) one obtains

$$m + i\dot{m}\frac{A}{\Omega(C - A)} - (\Delta\psi^c + \Delta\psi^{rd} + \Delta\psi^0) = \psi'$$

The right-hand side of this equation will be denoted as $\psi'$, i.e.

$$\psi' = \psi - (\Delta\psi^c + \Delta\psi^{rd} + \Delta\psi^0) \tag{12}$$

and it will be assumed, for the moment, that $\psi'$ (excitation independent of $m$ and $\dot{m}$) is equal to zero, so that one obtains the following equation

$$m\left[1 - \xi - \frac{k}{k_s}(1 - iQ^{-1})\right] + i\dot{m}\left[\frac{A_m}{\Omega(C - A)} + \frac{k}{k_s\Omega}(1 - iQ^{-1})\right] = 0\,, \tag{13}$$

where $A_m$ is the equatorial inertia momentum of the mantle, i.e. $A_m = A - A_c$.

The solution of equation (13) will be sought in the following form

$$m = m_0\, e^{(\alpha + i\sigma)t}\,, \tag{14}$$

expressing the damped circular motion in the direct sense since

$$\dot{m} = m_0(\alpha + i\sigma)\, e^{(\alpha + i\sigma)t}\,. \tag{15}$$

By substituting expressions (14) and (15) in Eq. (13) one obtains

$$1 - \xi - \frac{k}{k_s}(1 - \mathrm{i}Q^{-1}) + (\mathrm{i}\alpha - \sigma)\left[\frac{A_m}{\Omega(C - A)} + \frac{k}{k_s\Omega}(1 - \mathrm{i}Q^{-1})\right] = 0\,.$$

In this equation both parts should be equal to zero: the real part

$$1 - \xi - \frac{k}{k_s} - \frac{\sigma}{\Omega}\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) + \frac{\alpha}{\Omega}\frac{k}{k_s}Q^{-1} = 0$$

and the imaginary part

$$\frac{k}{k_s}Q^{-1} + \frac{\alpha}{\Omega}\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) + \frac{\sigma}{\Omega}\frac{k}{k_s}Q^{-1} = 0\,.$$

These two equations yield the solutions for $\sigma$ and $\alpha$:

$$\sigma \doteq \Omega\left(1 - \xi - \frac{k}{k_s}\right)\Big/\left(\frac{A_m}{C - A} + \frac{k}{k_s}\right) \tag{16}$$

$$\alpha \doteq -\frac{k}{k_s}\Omega Q^{-1}\Big/\left(\frac{Am}{C - A} + \frac{k}{k_s}\right),$$

since $(k/k_s)\,Q^{-1}$ is very small compared to $A_m/(C - A)$.

By using the following numerical values (Vondrák 1984):

$$\frac{k}{k_s} = 0{\cdot}3173$$

$$\frac{A_m}{(C - A)} = 269{\cdot}66$$

$$\Omega = 6{\cdot}3004 \text{ rad day}^{-1}$$

one obtains

$$\sigma = 0{\cdot}01593 - 0{\cdot}0233\xi \text{ rad day}^{-1}\,, \tag{17}$$

$$\alpha = -0{\cdot}0074Q^{-1} \text{ day}^{-1}$$

where the factor $\xi$ describes the dependence of $\sigma$ on the amplitude $A$. If the expression yielding $\xi$ from equation (9) is substituted in the first Eq. (17), the following value is then obtained for Chandler's frequency:

$$\sigma = 0{\cdot}01444 + 0{\cdot}00149\,\mathrm{e}^{-20A}\,, \tag{18}$$

This is a fine approximation for the plot presented in Fig. 3 (Vondrák 1988).

If instead of the zero on the right-hand side of equation (13) the excitation $\psi'$ (independent of $m$) is inserted, one finds that

$$m + \mathrm{i}\frac{\dot{m}}{(\sigma - \mathrm{i}\alpha)} = \frac{\psi'}{1 - \xi - (k/k_s)(1 - \mathrm{i}Q^{-1})}\,, \tag{19}$$

where $\sigma$ is not constant. This differential equation has a solution:

$$m = \exp\left[\textstyle\int_0^t(\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau\right]\left[m_0 - \frac{(\alpha + \mathrm{i}\sigma)}{1 - \xi - (k/k_s)(1 - \mathrm{i}Q^{-1})}\int_0^t \psi' \exp\left\{-\textstyle\int_0^t(\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau\right\}\mathrm{d}\tau\right]$$

or

$$m = \exp\left[\textstyle\int_0^t(\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau\right]\left[m_0 - \frac{\mathrm{i}\Omega}{A_m/(C - A) + k/k_s}\int_0^t \psi' \exp\left\{-\textstyle\int_0^t(\alpha + \mathrm{i}\sigma)\,\mathrm{d}\tau\right\}\mathrm{d}\tau\right]. \tag{20}$$

## 5. Atmospheric Excitation

The equatorial effective angular momentum functions (EAMF) in a complex notation are (Barnes et al. 1983)

$$\chi = \chi_1 + i\chi_2 = \frac{1{\cdot}00R^4}{g(C_m - A_m)} \int_s p_s e^{i\lambda} \sin\phi \cos^2\phi d\lambda d\phi - $$
$$- \frac{1{\cdot}43R^3}{g\Omega(C_m - A_m)} \int_v (u \sin\phi + iv) e^{i\lambda} \cos\phi \, d\lambda \, d\phi \, dp . \tag{21}$$

The integral with subscript $s$ is a surface integral integrated over the Earth's surface, the one with subscript $v$ is a volume integral integrated over the atmosphere; $\phi$, $\lambda$ and $p_s$ are geographic latitude, longitude and the air pressure on the Earth's surface, respectively; $u$ and $v$ are the wind components ($u$ the eastward one and $v$ the northward). The first term of equation (21) is the pressure term and the second is the wind term; they express the influence of the redistribution of the air mass and its relative angular momentum, respectively. At some meteorological centres the pressure term is calculated with the inverted barometer correction.

Because of differences between the present Earth model and Barnes's (1983) it is necessary to express the excitation functions (Eq. 12) by means of Barnes's equatorial angular momentum functions of the atmosphere ($\chi_1$, $\chi_2$).

Expression (21) can be written in the form (Vondrák 1987):

$$\chi = \chi_1 + \chi_2 = (\chi_1^p + i\chi_2^p) + (\chi_1^w + i\chi_2^w) = E_p I^p + E_w I^w , \tag{22}$$

where $I^p$ and $I^w$ are a surface and a volume integral, respectively, and $E_p$ and $E_\omega$ are numerical constants.

The excitation functions $\psi'$ (Eq. 12) will be calculated using Vondrák's (1987) numerical values.

Because of the different values of the numerical constants $E_p$ and $E_w$ used by Barnes (1983) and by Vondrák (1987), as well as because of multiplying only the pressure term of excitation functions (2) by $1 + k_2 = 0{\cdot}70$ (due to the surface loading deformation)

$$\psi' = 0{\cdot}628 \left( \chi - i \frac{\dot{\chi}}{\Omega} \right) \tag{23}$$

## 6. Model of the Earth with Variable Chandler's Frequency

By introducing a new excitation function (11) for the nonequilibrium ocean dependent on the polar motion amplitude, the equations of the elastic Earth model with variable Chandler's frequency are derived.

The coordinates of the excitation pole ($x$, $y$) as functions of the atmospheric equatorial EAMF are obtained by substituting the first equation (23) into (20)

$$m = \exp\left[\int_0^t (\alpha + i\sigma) d\tau\right] \left[ m_0 - \frac{i\Omega}{A_m/(C - A) + k/k_s} 0{\cdot}628 \int_0^t (\chi - i\dot{\chi}/\Omega) \exp\left\{-\int_0^t (\alpha + i\sigma) d\tau\right\} d\tau \right]. \tag{24}$$

Since $\sigma$ is variable (it is given by Eq. 18), one obtains for the integral in the exponent of (24)

$$\int_0^t [\alpha + i\,\sigma(\tau)] \, d\tau = \int_0^t [\alpha + i(\sigma_0 + 0{\cdot}00149 \, e^{-20A(\tau)})] \, d\tau ,$$

i.e.

$$\int_0^t [\alpha + i\sigma(\tau)] \, d\tau = \alpha t + i(\sigma_0 t + J) ,$$

where

$$J = 0{\cdot}00149 \int_0^t e^{-20A(\tau)} \, d\tau \tag{25}$$

and

$$\sigma = \sigma_0 + 0{\cdot}00149 \, e^{-20A(t)} .$$

If for reasons of simplicity one writes $l = 0{\cdot}628$ and $L = A_m/(C - A) + k/k_s$, equation (24) becomes

$$m = e^{\alpha t + i(\sigma_0 t + J)} \{ m_0 - (i\Omega l/L) [\int_0^t \chi \, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} \, d\tau - (i/\Omega) \int_0^t \dot{\chi} \, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} \, d\tau]\}. \tag{26}$$

Integrating by parts the second integral in equation (26) one finds that

$$\int_0^t \dot{\chi} \, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} \, d\tau = (\alpha + i\sigma) \int_0^t \chi \, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} \, d\tau + \chi(t) \, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} - \chi(0) .$$

By substituting this expression into equation (26) one obtains

$$m = e^{-[\alpha\tau + i(\sigma_0\tau + J)]} \{m_0 - (l/L)[\alpha + i(\Omega + \sigma)] \int_0^t \chi\, e^{-[\alpha\tau + i(\sigma_0\tau + J)]} d\tau\} - (l/L)\{\chi(t) - \chi(0)\, e^{[\alpha\tau + i(\sigma_0\tau + J)]}\}$$

An extended version of this expression is

$$m_1 + im_2 = e^{\alpha t}[\cos(\sigma_0 t + J) + i \sin(\sigma_0 t + J)]$$
$$\{[m_1(0) + im_2(0)] - (l/L)([\alpha + i(\Omega + \sigma)] \int_0^t (\chi_1 + i\chi_2)\, e^{-\alpha\tau}[\cos(\sigma_0\tau + J) - i \sin(\sigma_0\tau + J)]\, d\tau\}$$
$$- (l/L)\{\chi_1(t) + i\,\chi_2(t) - [\chi_1(0) + i\,\chi_2(0)]\, e^{\alpha\tau}[\cos(\sigma_0 t + J) + i \sin(\sigma_0 t + J)]\}$$

This yields the coordinates of the excited pole in the normally oriented coordinate system in which the $x$ axis is directed along the zero meridian and the $y$ axis is positive eastwards. We neglected the factor $\alpha$ since it is 4 ~ 5 orders smaller than $\Omega$.

$$x = m_1(t) = e^{\alpha\tau}\{x_0 \cos(\sigma_0 t + J) - y_0 \sin(\sigma_0 t + J) +$$
$$+ (l/L)\,\Omega(1 + \sigma_t/\Omega)[(J_1 + J_4)\sin(\sigma_0 t + J) + (J_2 - J_3)\cos(\sigma_0 t + J)]\}$$
$$y = m_2(t) = e\{^{\alpha\tau}x_0 \sin(\sigma_0 t + J) + y_0 \cos(\sigma_0 t + J) -$$
$$- (l/L)\,\Omega(1 + \sigma_t/\Omega)[(J_1 + J_4)\cos(\sigma_0 t + J) - (J_2 - J_3)\sin(\sigma_0 t + J)]\}\,. \quad (27)$$

The quantities designated $J_1$, $J_2$, $J_3$ and and $J_4$ are:

$$J_1 = \int_0^t \chi_1\, e^{-\alpha\tau}\cos(\sigma_0\tau + J)\, d\tau + \Omega^{-1}[\chi_2(t)\, e^{-\alpha\tau}\cos(\sigma_0 t + J) - \chi_2(0)]$$
$$J_2 = \int_0^t \chi_2\, e^{-\alpha\tau}\cos(\sigma_0\tau + J)\, d\tau - \Omega^{-1}[\chi_1(t)\, e^{-\alpha\tau}\cos(\sigma_0\tau + J) - \chi_1(0)]$$
$$J_3 = \int_0^t \chi_1\, e^{-\alpha\tau}\sin(\sigma_0\tau + J)\, d\tau + \Omega^{-1}\,\chi_2(t)\, e^{-\alpha\tau}\sin(\sigma_0 t + J)$$
$$J_4 = \int_0^t \chi_2\, e^{-\alpha\tau}\sin(\sigma_0\tau + J)\, d\tau - \Omega^{-1}\,\chi_1(t)\, e^{-\alpha\tau}\sin(\sigma_0 t + J) \quad (28)$$

Equation (27) yields the coordinates of the excited pole $x$, $y$ resulting from the model of the Earth with a nonequilibrium ocean. Substituting the numerical value of $l$ and the expression for $L$ into (27) one has

$$x = e^{\alpha\tau}\{x_0 \cos\sigma_0 t + J) - y_0 \sin(\sigma_0 t + J) +$$
$$+ \frac{0{\cdot}628}{A_m/(C - A) + k/k_s}\,\Omega(1 + \sigma/\Omega)[(J_1 + J_4)\sin(\sigma_0 t + J) + (J_2 - J_3)\cos(\sigma_0 t + J)]\}$$
$$y = e^{\alpha\tau}\{x_0 \sin(\sigma_0 t + J) + y_0 \cos(\sigma_0 t + J) -$$
$$- \frac{0{\cdot}628}{A_m/(C - A) + k/k_s}\,\Omega(1 + \sigma/\Omega)[(J_1 + J_4)\cos(\sigma_0 t + J) - (J_2 - J_3)\sin(\sigma_0 t + J)]\} \quad (29)$$

In this way the response of the Earth's model with a variable and amplitude-dependent Chandler's frequency to the atmospheric excitation is derived. This model was confirmed on the basis of a comparison with observations (Vondrák and Pejović 1989) even on a short-period scale (1976–1986).

## 7. Conclusions

The hypothesis of nonlinear dependence of Chandler's frequency on the polar motion total amplitude is confirmed by analysig the ILS pole coordinates. The response of the Earth to atmospheric excitation is derived by using an axially symmetric model of the Earth with a variable and amplitude-dependent Chandler's frequency. This model of the Earth is confirmed by comparing it to observations, even on a short-period scale. In this way the hypothesis of a variable Chandler's frequency due to the influence of the nonequilibrium ocean on polar motion is confirmed.

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