



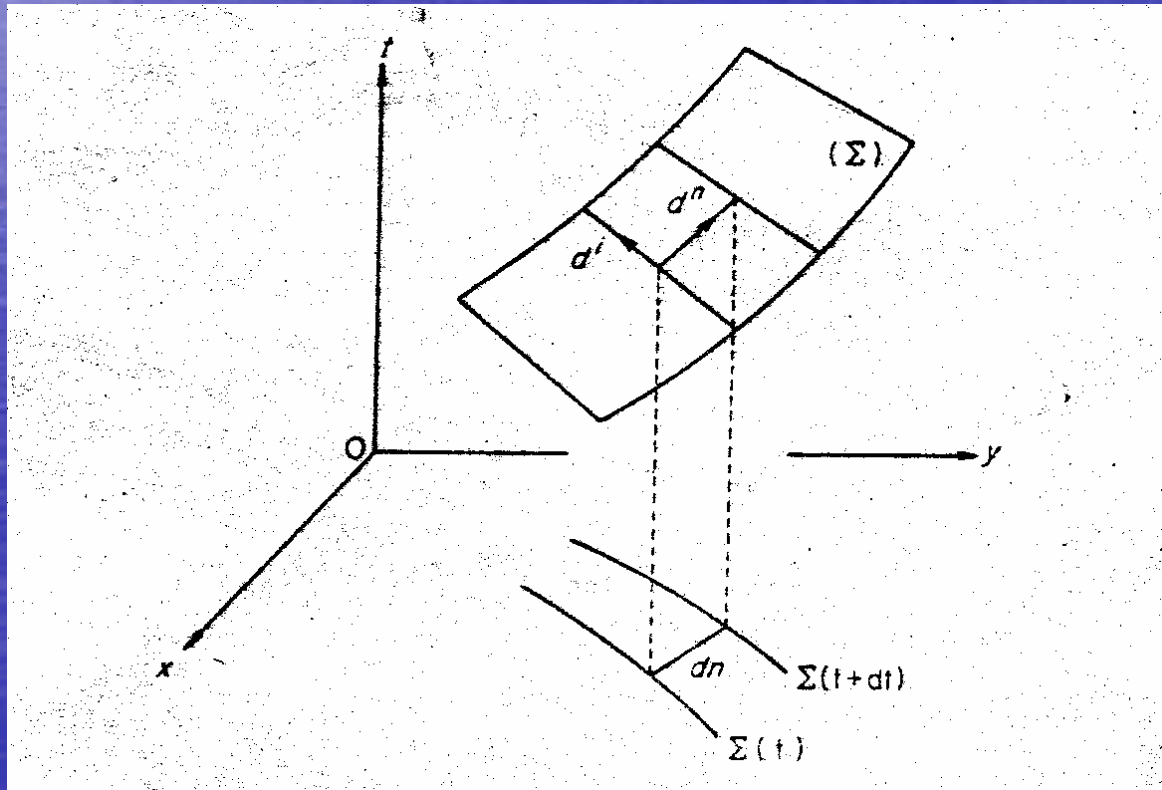
Dejan Urošević

Department of Astronomy, Faculty of Mathematics,
University of Belgrade

Shock Waves: I. Brief Introduction +
Supernova Remnants

Shock Waves - Introduction

- Mathematical description
 - Three dimensional manifold (Σ) in four dimensional space (t, x, y, z)
 - $\Sigma(t)$ – surfaces in three dimensional Euclidean space



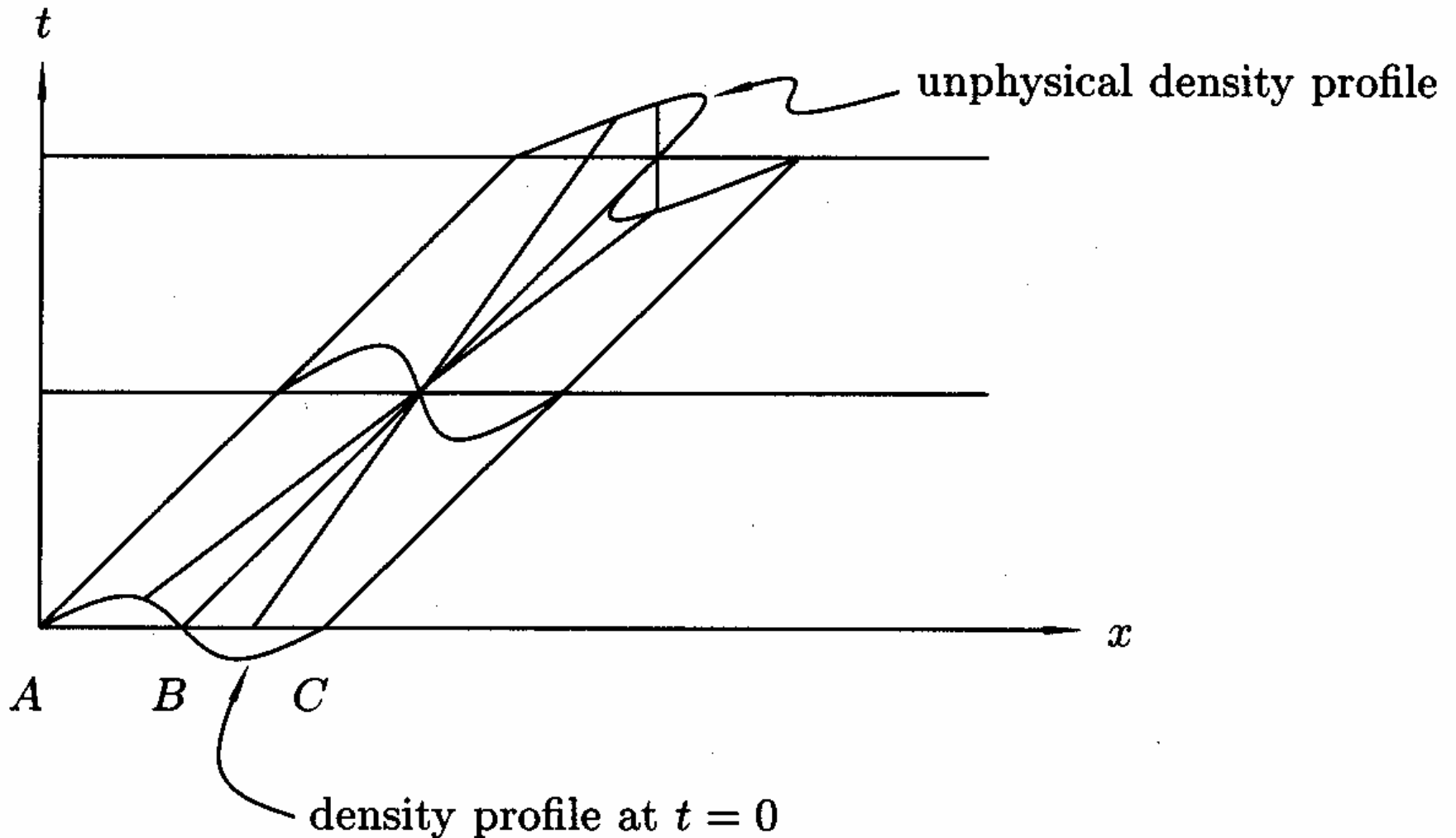
Shock Waves - Introduction

- Cauchy's problem (finding solution of MHD equations for H, v, p, ρ on (Σ))
- When system is indeterminate $\Rightarrow (\Sigma)$ is so-called **characteristic manifold**
- Surface $\Sigma(t)$ associated with a char. manifold is **wave surface** or briefly **wave**
- If two different solutions of MHD equations take same value on $(\Sigma) \Rightarrow (\Sigma)$ is char. manifold
- But first derivatives on char. (Σ) have different values

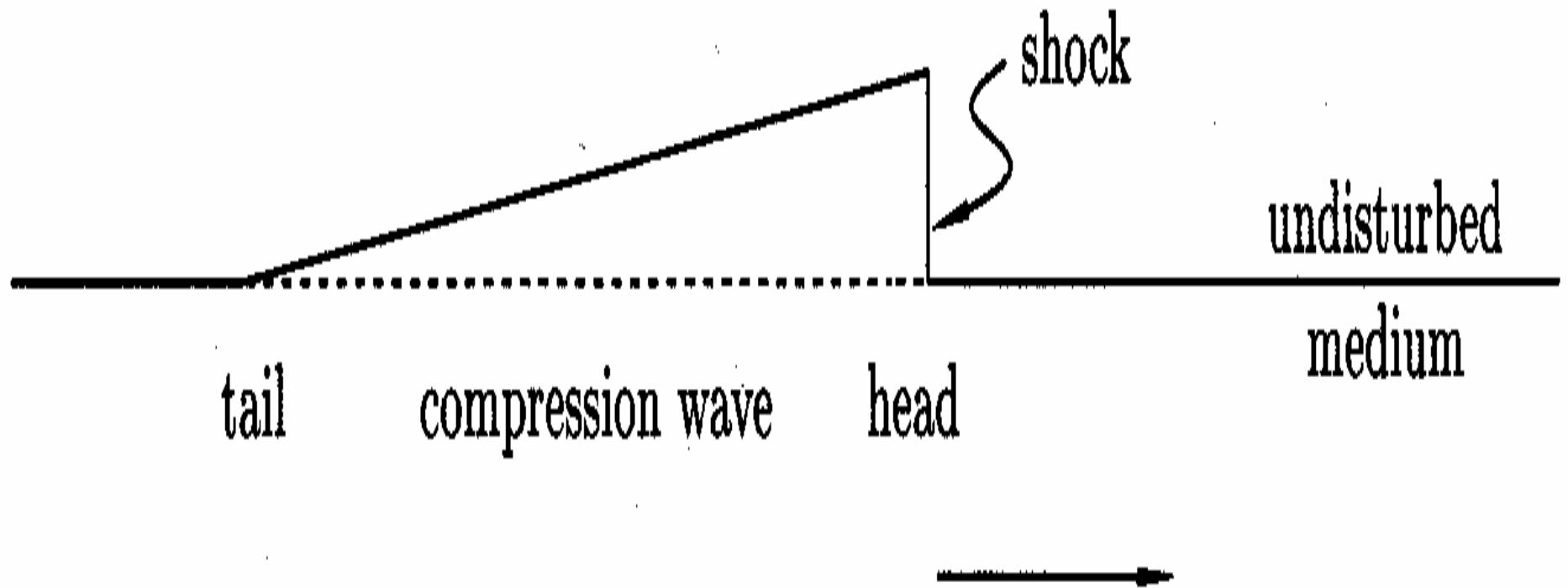
Shock Waves - Introduction

- Propagation of disturbances in fluids -> by waves
- Wave surface $\Sigma(t)$ represents border between disturbed and non-disturbed fluid
- If wave is appeared as steep (\sim vertical) in its profile it is **shock wave**

Shock Waves - Introduction



Shock Waves - Introduction



Shock Waves – HD equations

- Rankine – Hugoniot jump conditions
region 1 – upstream, region 2 - downstream

- conservation of mass

$$\rho_2 u_2 = \rho_1 u_1$$

- conservation of momentum

$$\rho_2 u_2^2 + p_2 = \rho_1 u_1^2 + p_1$$

- conservation of energy

$$1/2 u_2^2 + h_2 = 1/2 u_1^2 + h_1,$$

$h = \gamma/(\gamma-1) \cdot p/\rho$ – specific enthalpy

Shock Waves – HD equations

far downstream


u_2 

$\rho_2, P_2, T_2, \mathcal{E}_2$

(nearly) uniform gas

thin
transition
layer
effecting
shock
“jump”

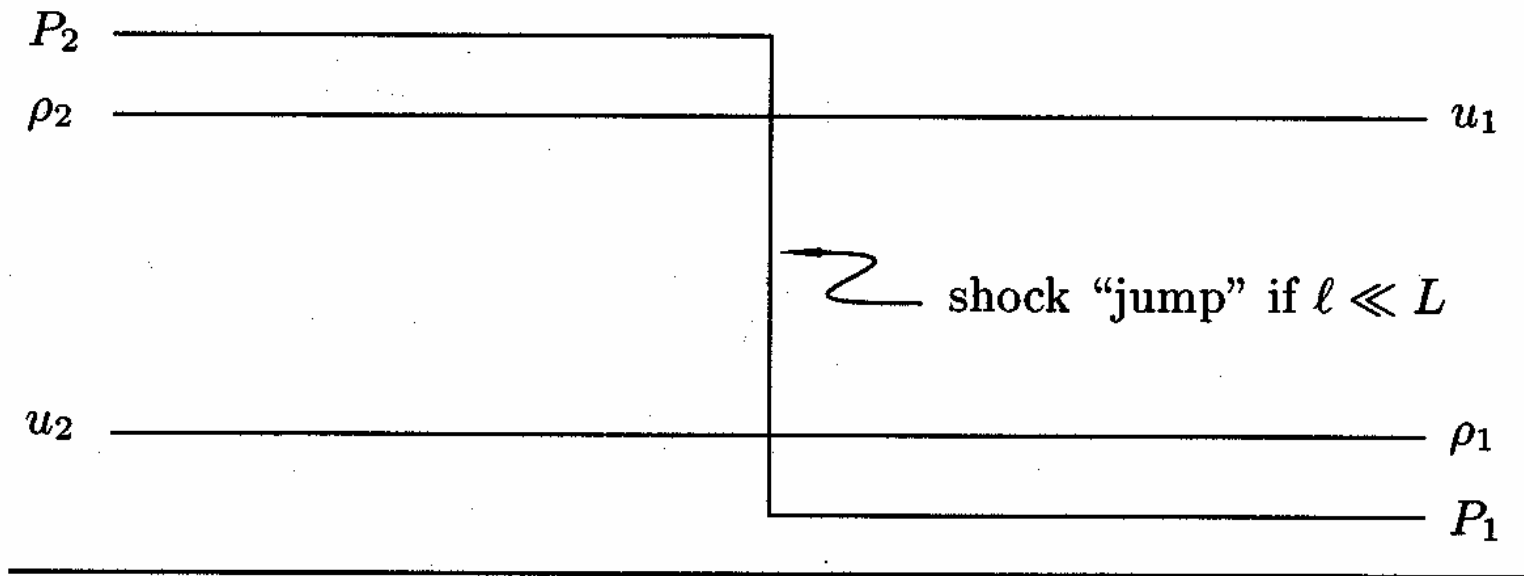
far upstream

u_1 

$\rho_1, P_1, T_1, \mathcal{E}_1$

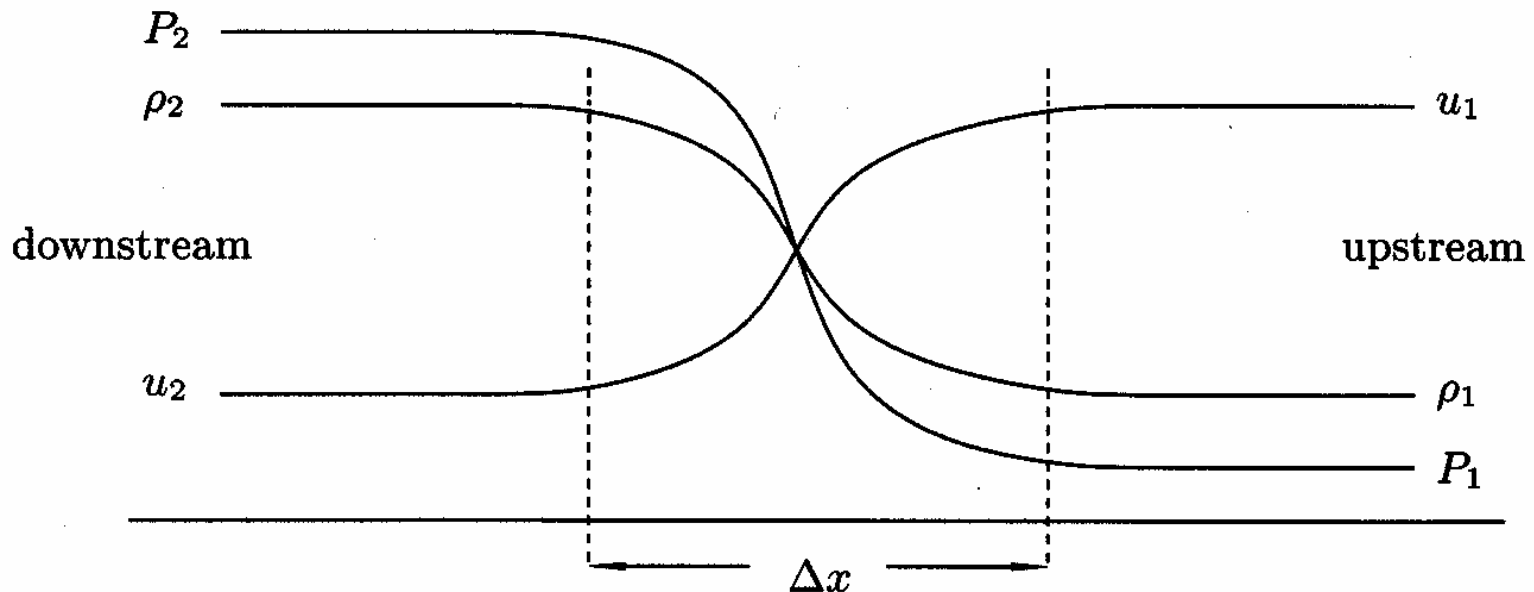
(nearly) uniform gas

Shock Waves – HD equations

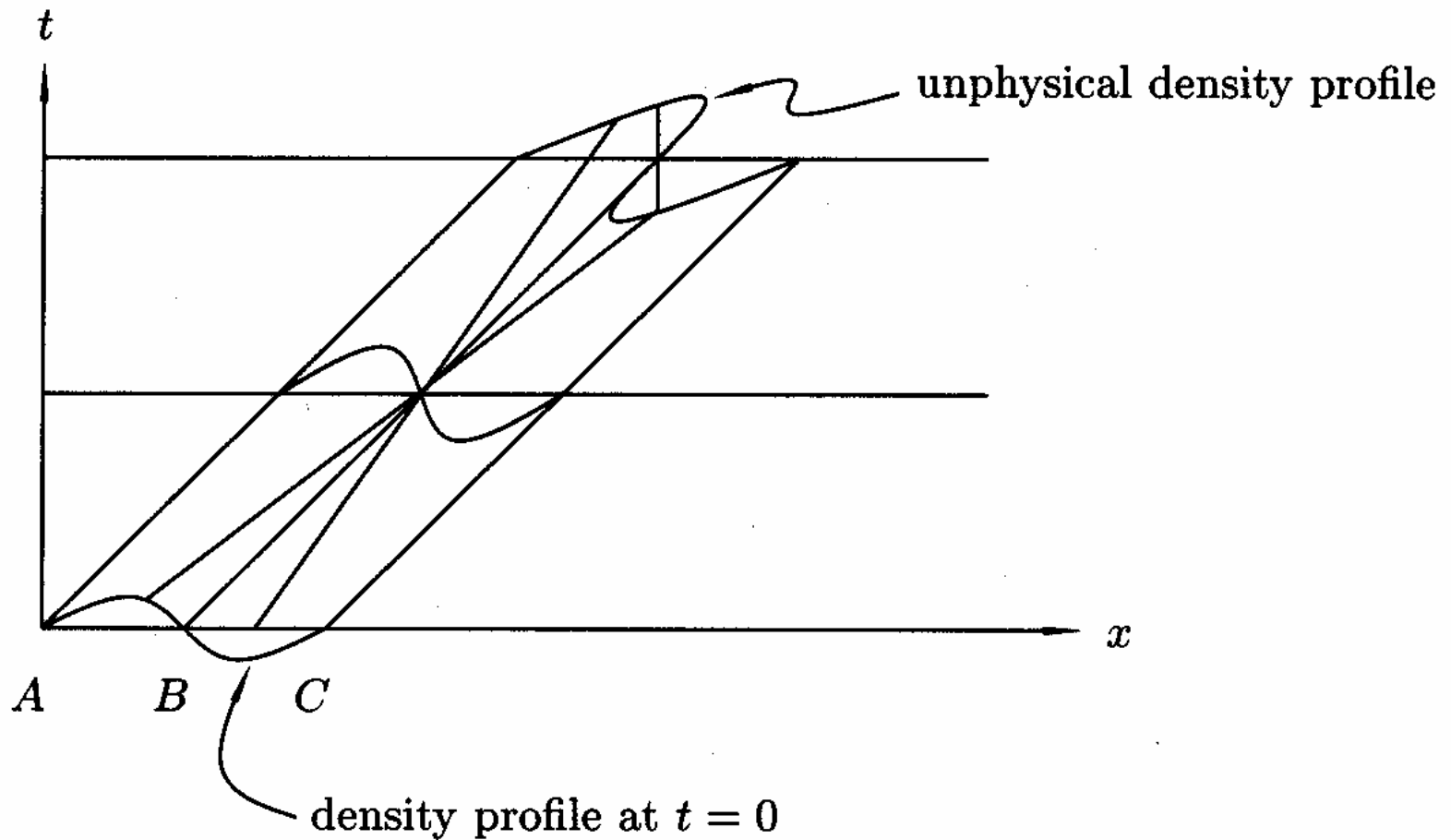


Shock Waves – Viscous Shocks

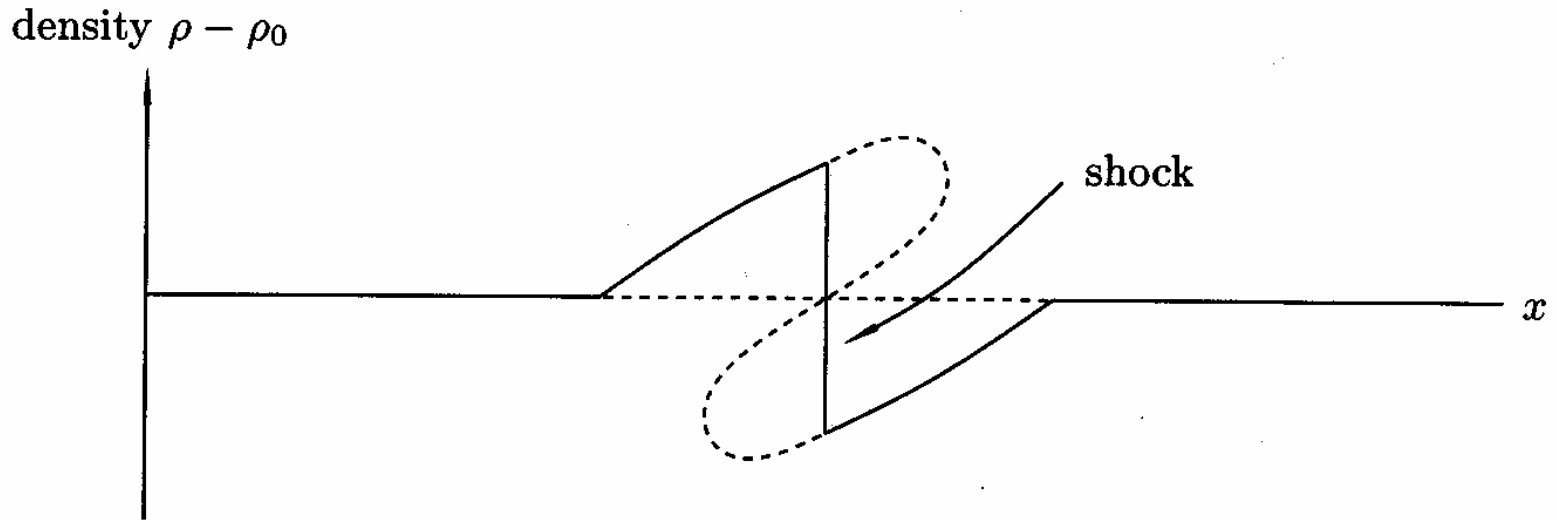
- $\Delta x \sim l$, l – mean free path for elastic collision



Steepening of Acoustic Waves Into Shock Waves



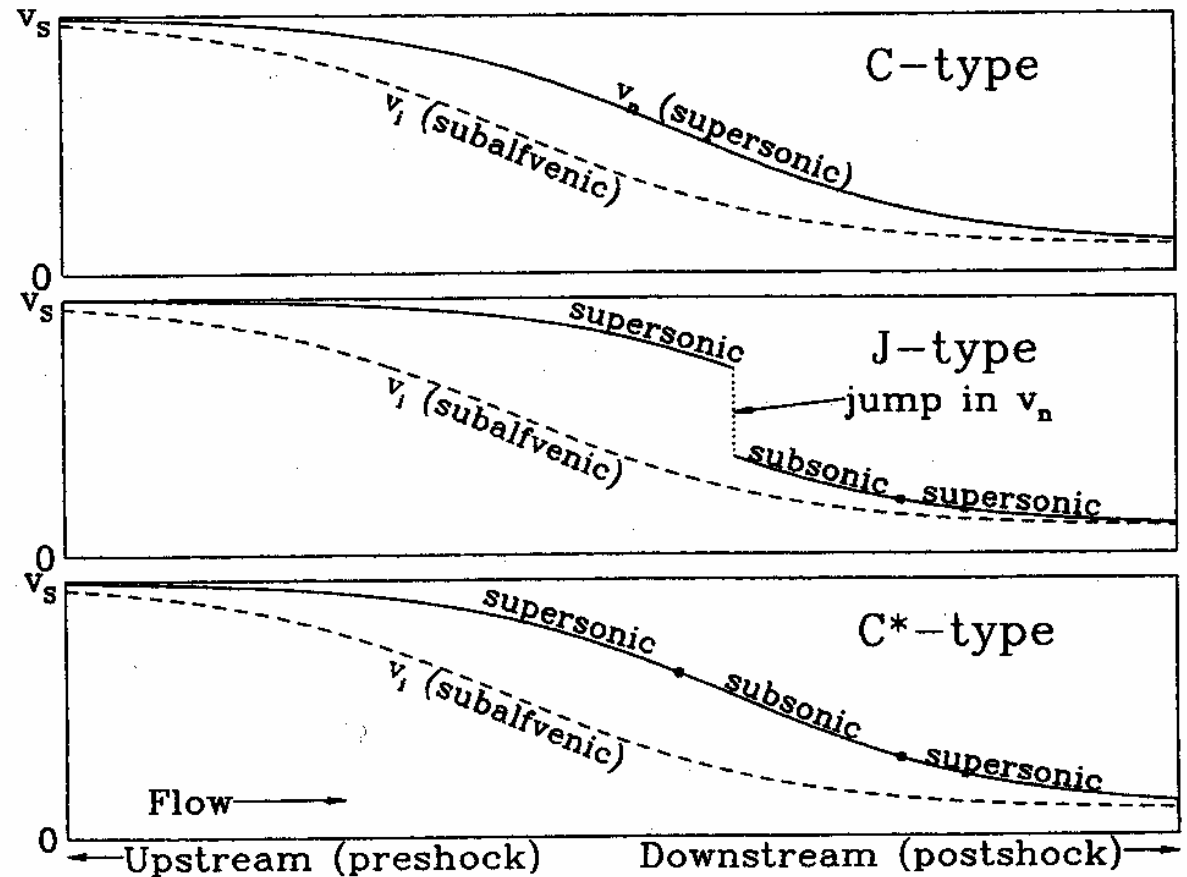
Steepening of Acoustic Waves Into Shock Waves

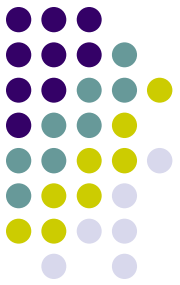




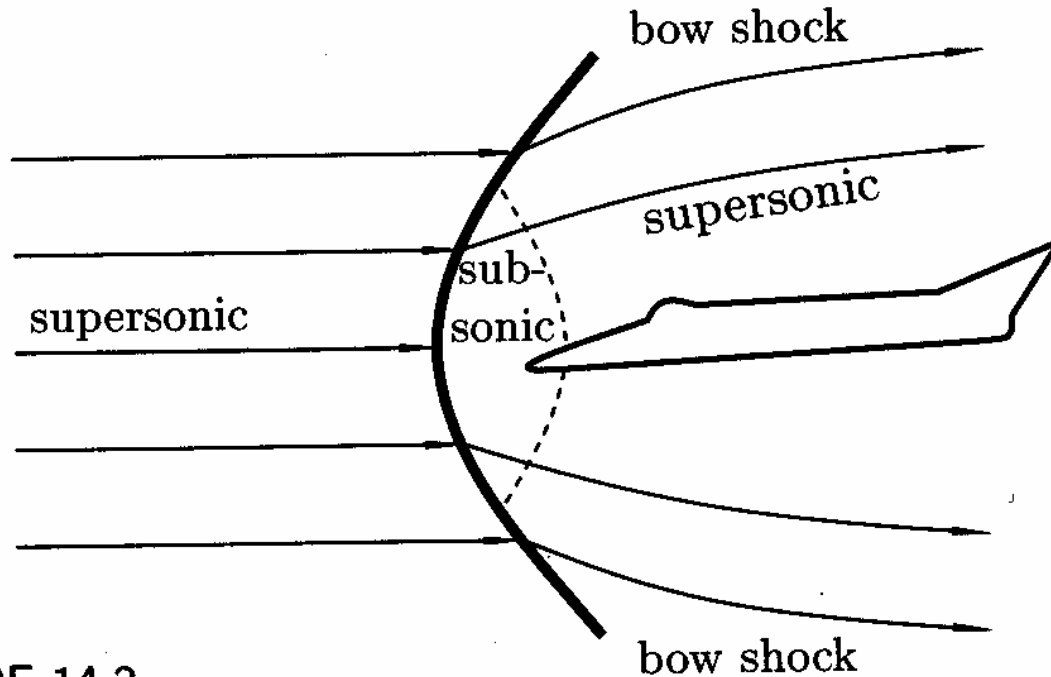
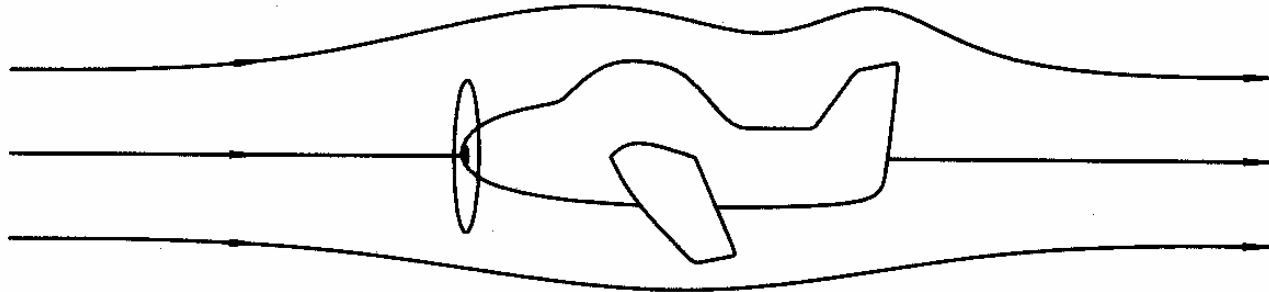
Types of shock waves

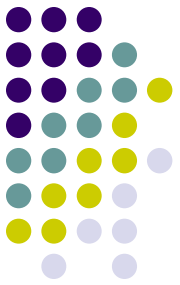
- C shocks (continuous)
- J shocks
- C* shocks





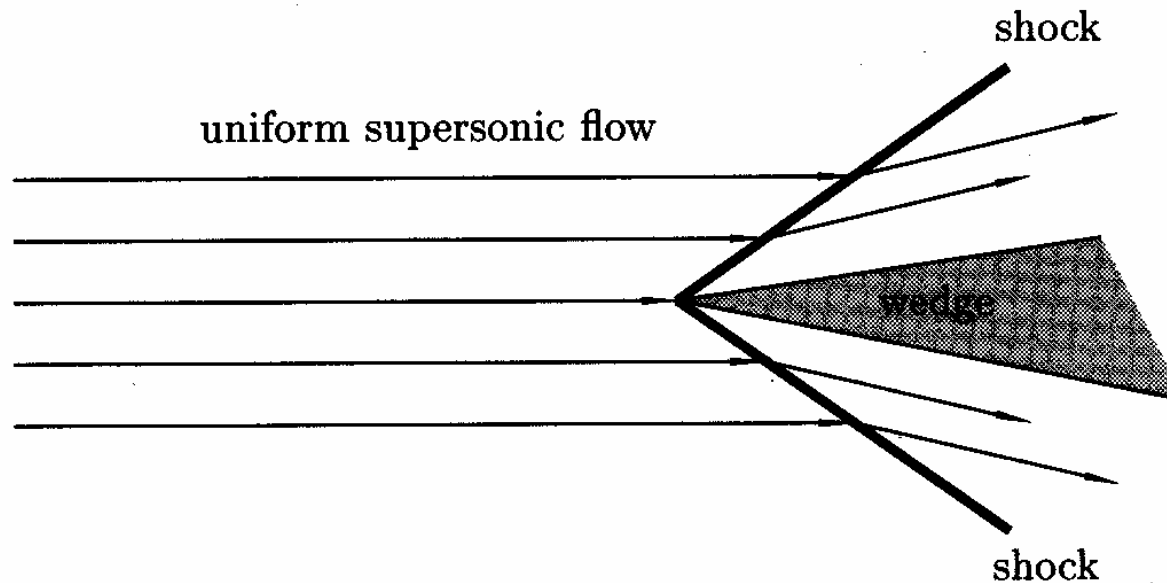
Types of shock waves



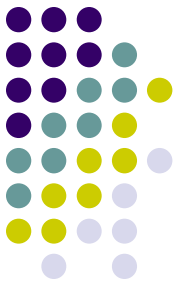


Types of shock waves

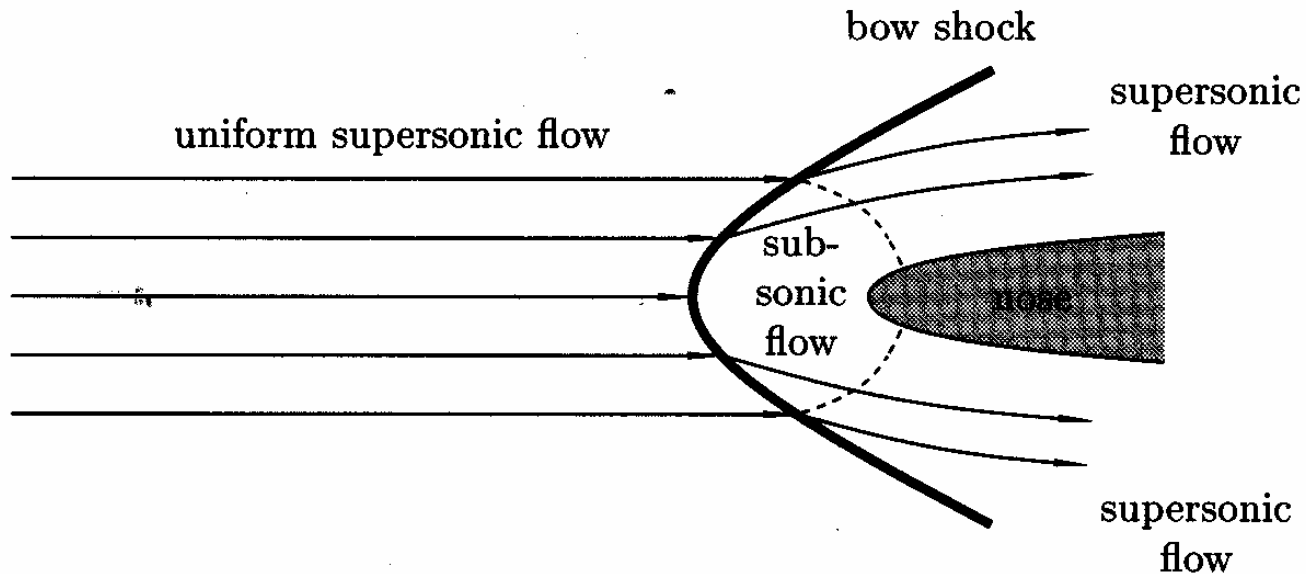
shocks on an infinite wedge with a small opening angle



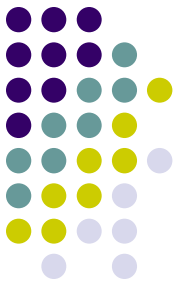
Types of shock waves



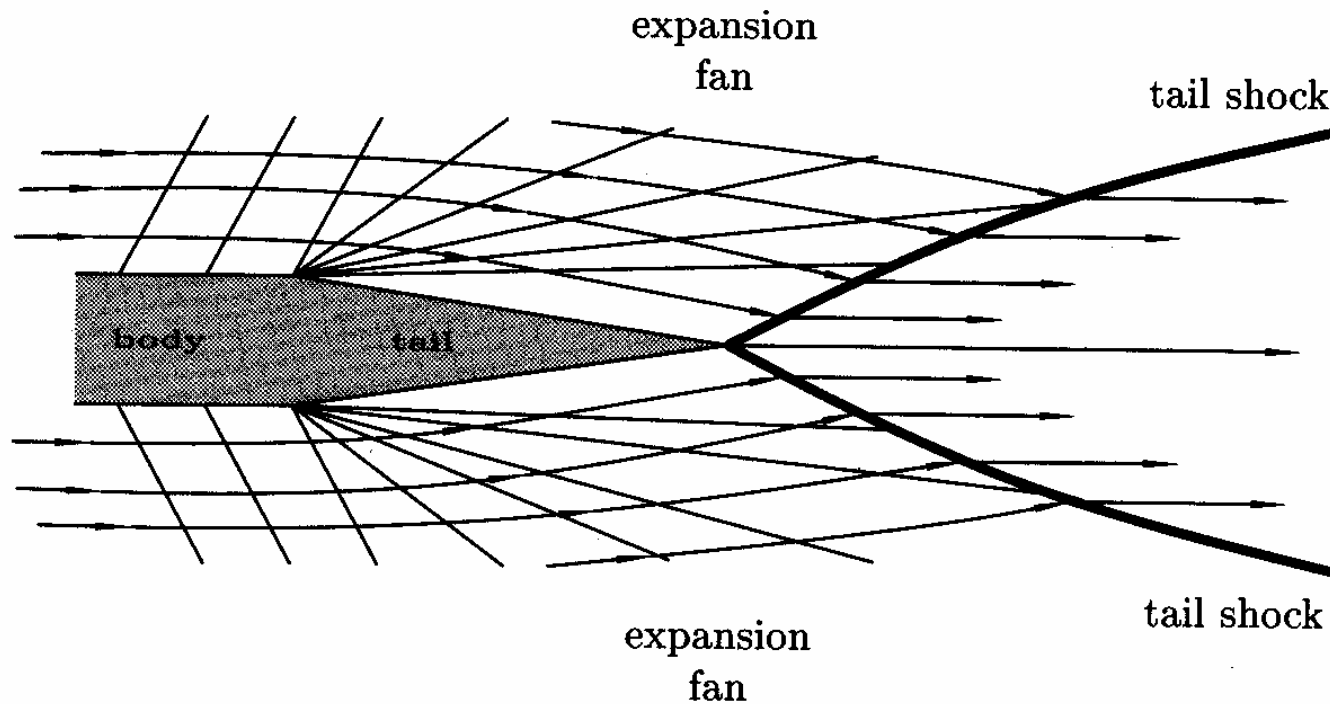
bow shock



Types of shock waves



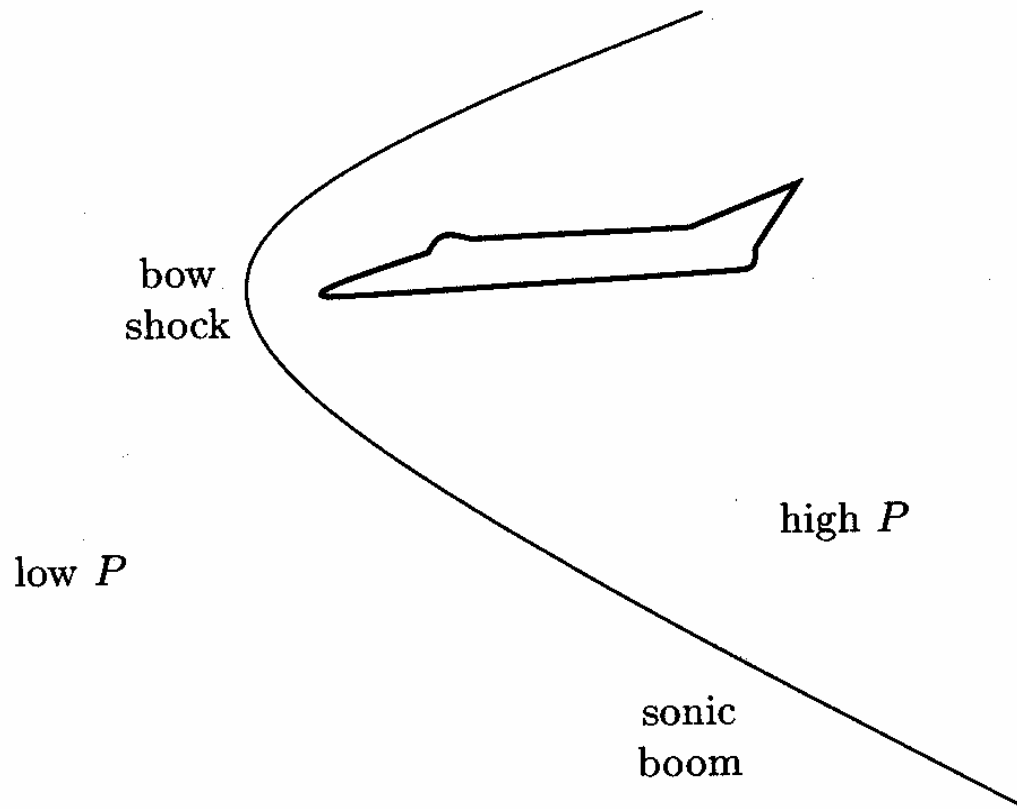
tail shock

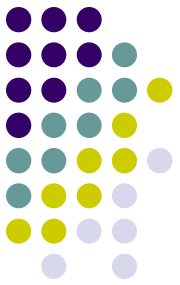




Types of shock waves

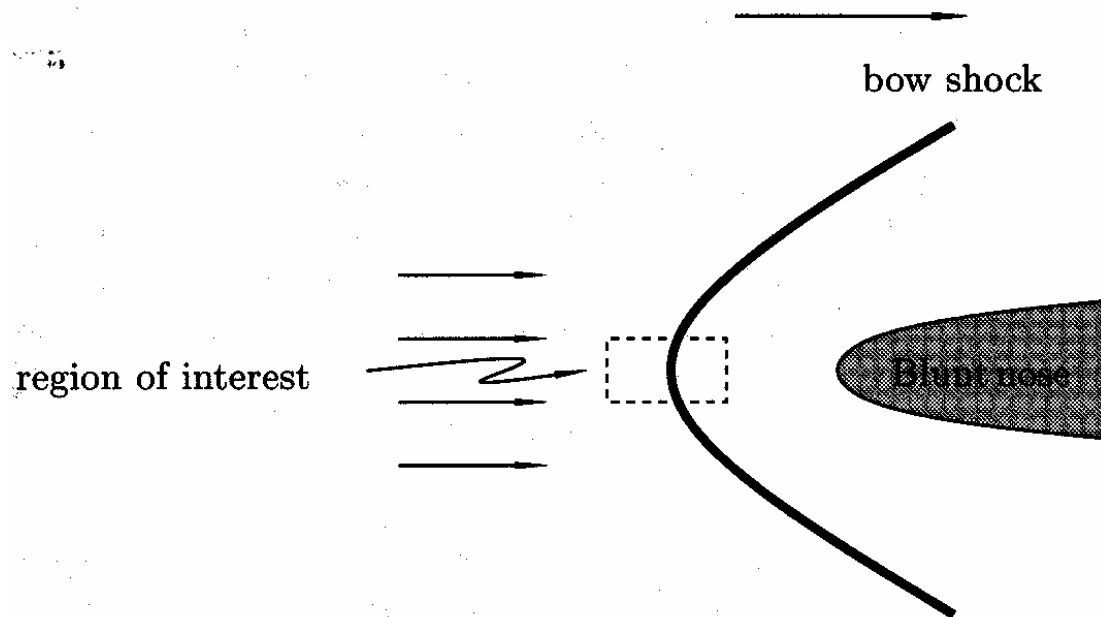
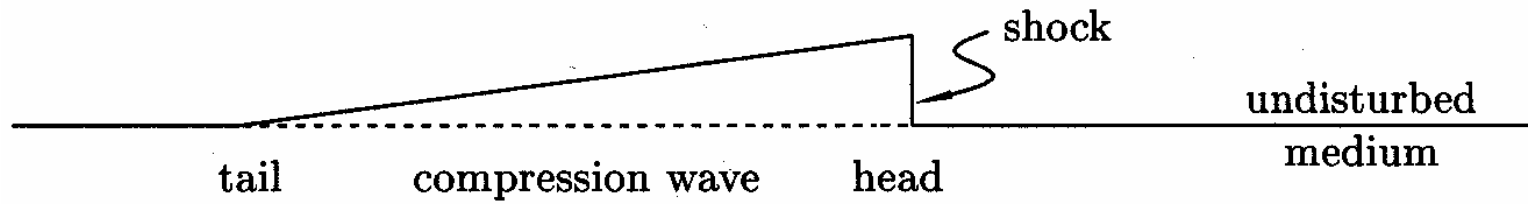
- farther away from the aircraft – shock wave degenerates to a acoustic wave

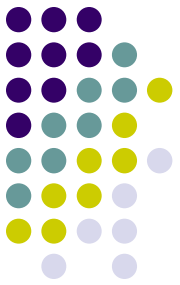




Types of shock waves

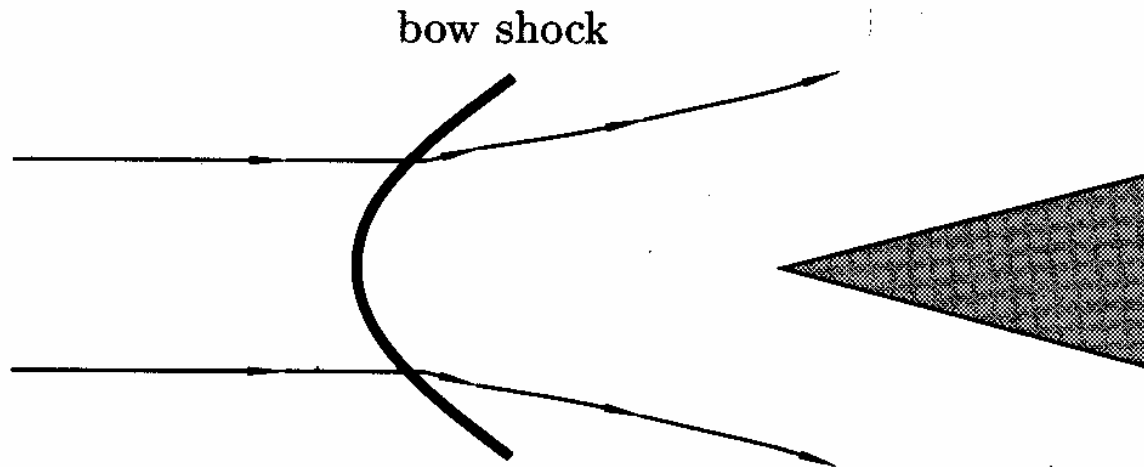
- bow shock ~ normal shock





Types of shock waves

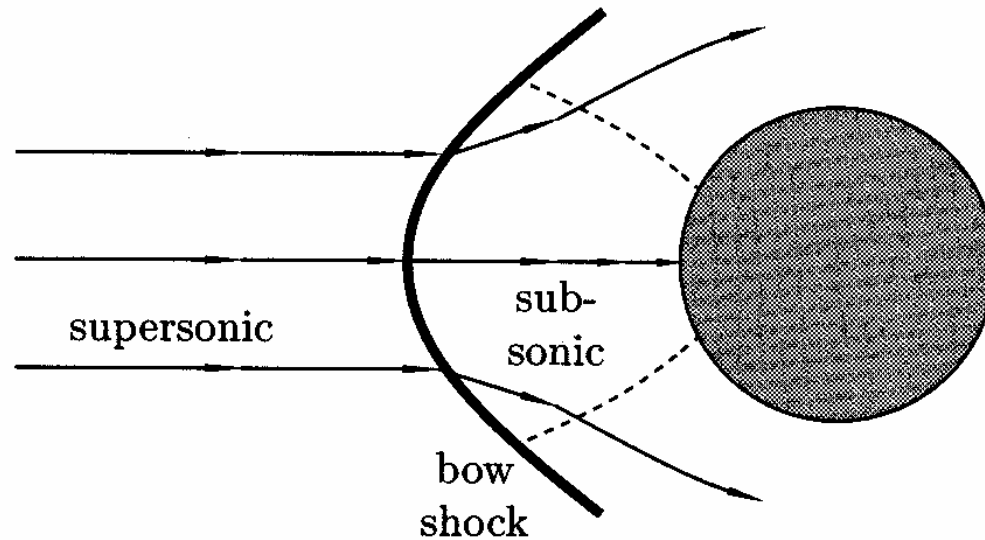
slow supersonic flow of a body with a sharp nose

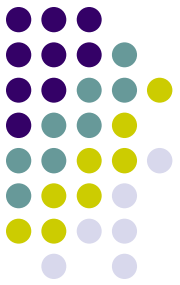




Types of shock waves

body with a blunt nose flying supersonically at any speed





Types of shock waves

- STRONG SHOCK:

jump in velocity

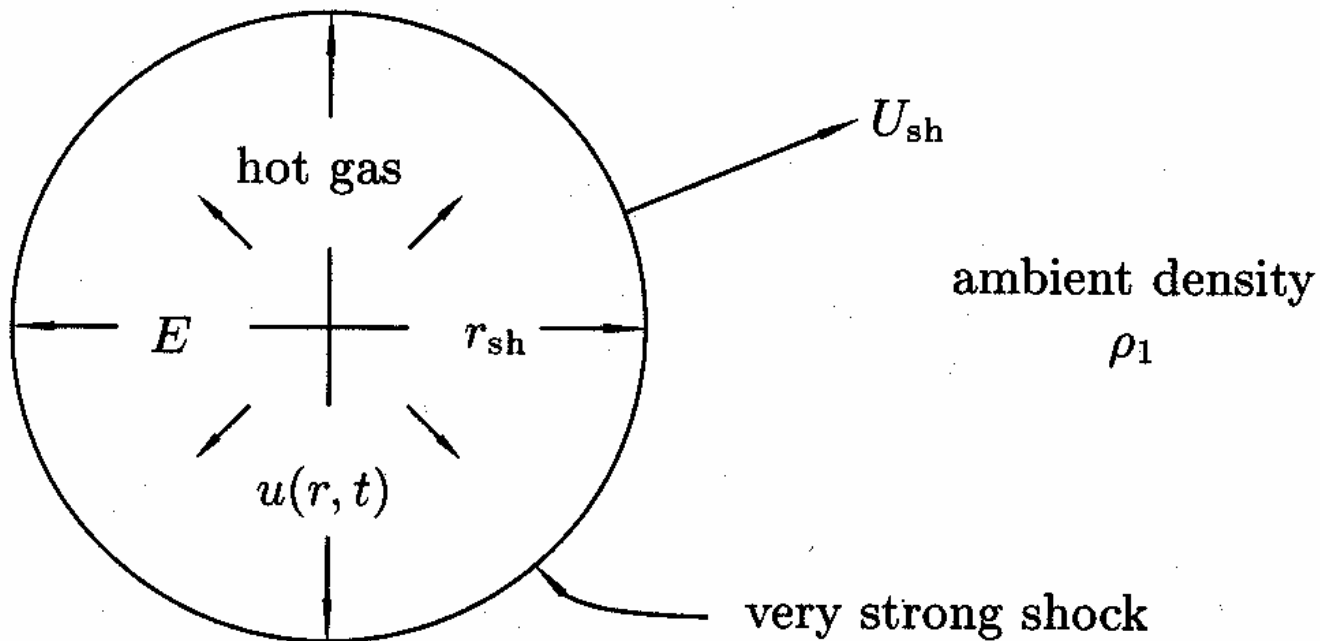
$$\Delta u = u_1 - u_2 \sim u_1$$



Blast Waves and Supernova Remnants

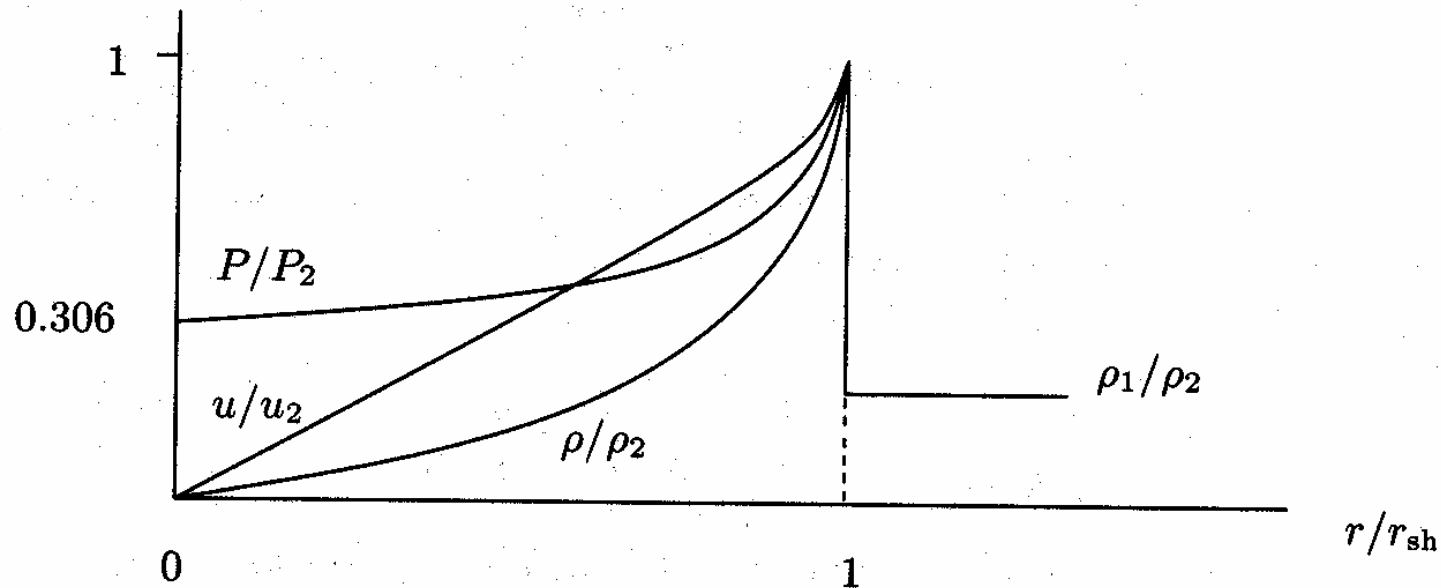
Blast Waves and Supernova remnants

- the point release of a large amount of energy creates a spherical blast wave



Blast Waves and Supernova remnants

- the blast wave solution - Sedov and Taylor $r \sim t^{2/5}$



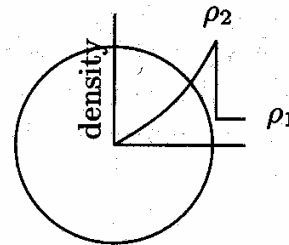
Blast Waves and Supernova remnants

- compression:

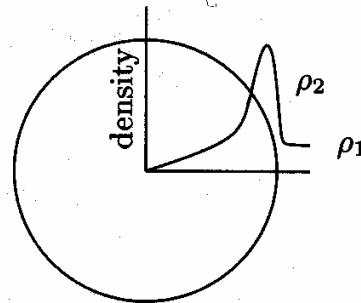
$$\rho_2/\rho_1=4$$

HD phases of an SNR evolution

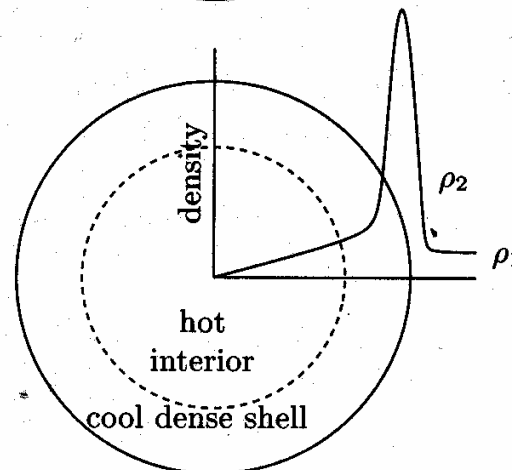
blast wave
(energy conserving)



shell formation
(radiative losses $\lesssim E$)

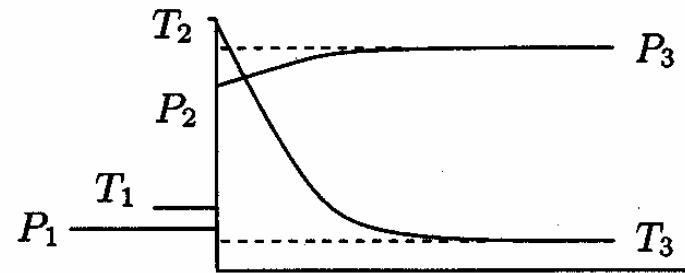
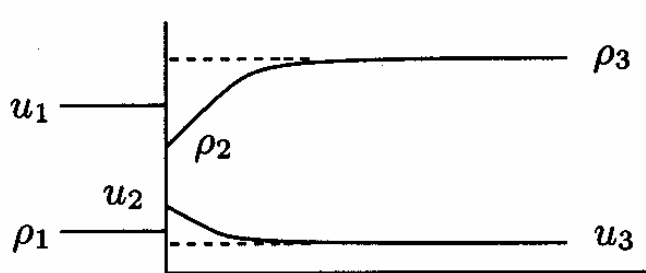
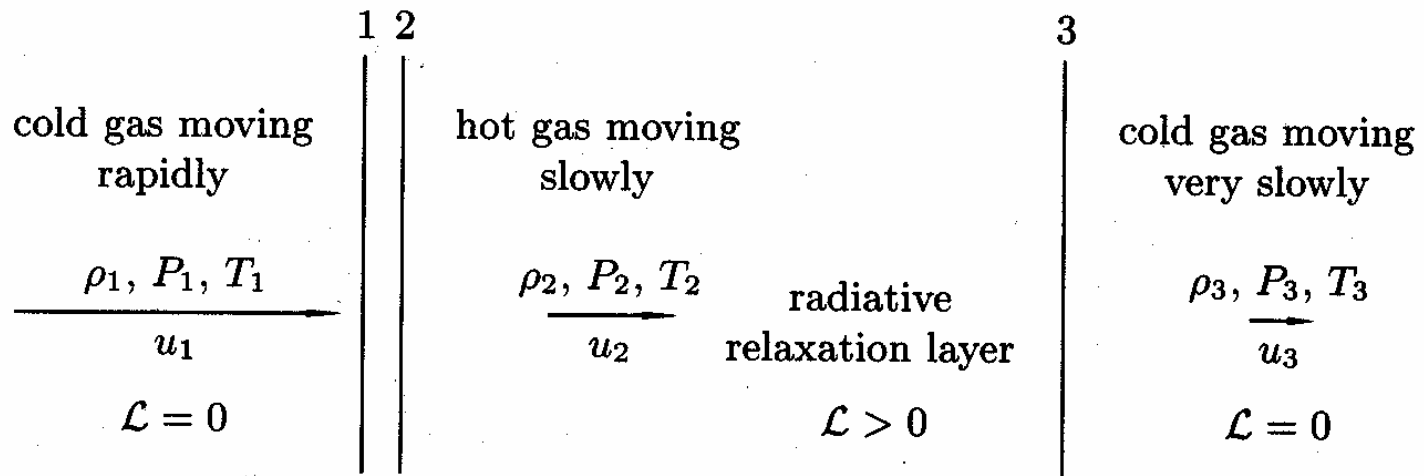


snow plow
(momentum conserving)



HD phases of SNR evolution

■ radiative shocks



HD phases of SNR evolution

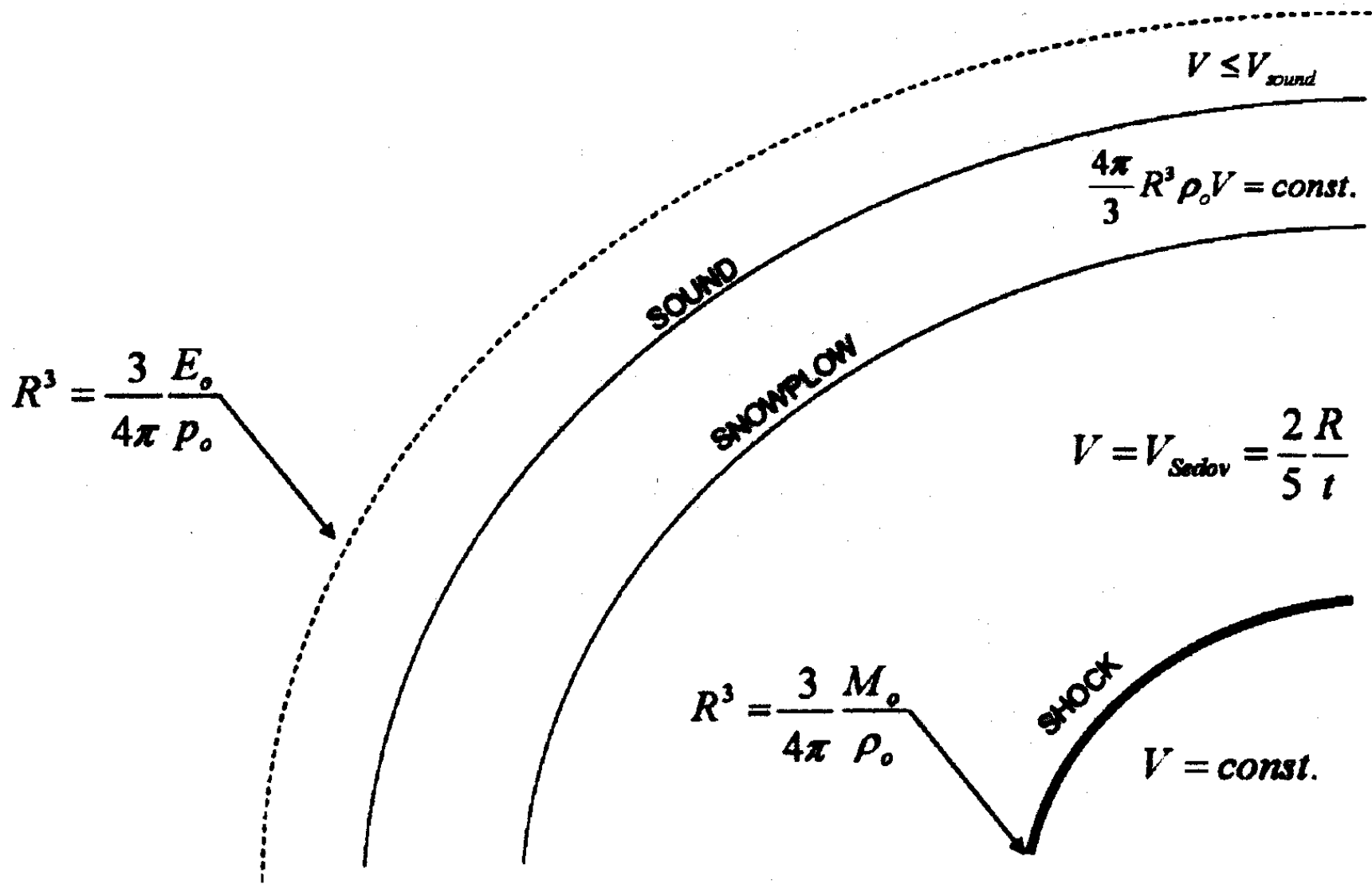
- isothermal shock
compression:

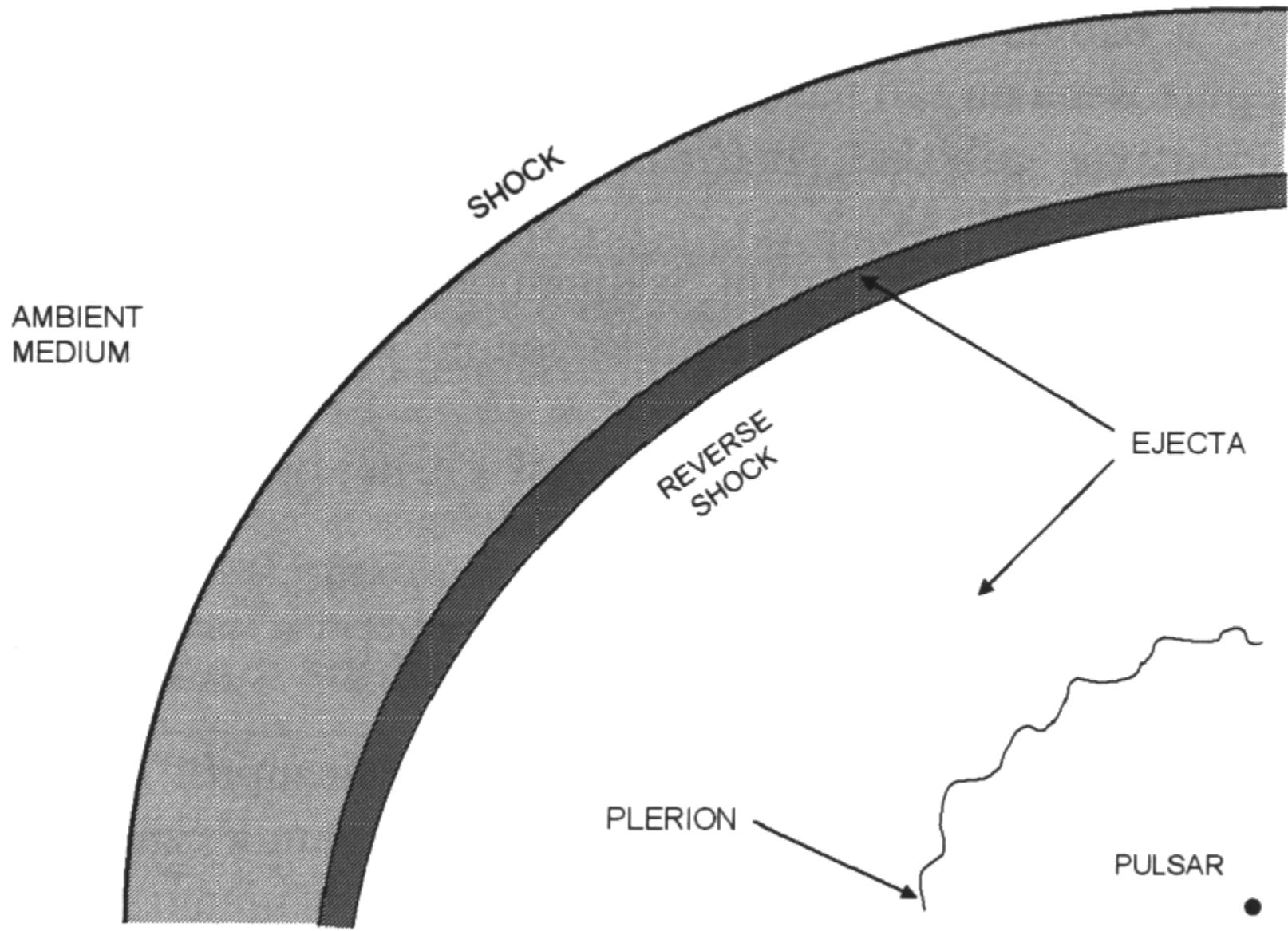
$$\rho_2/\rho_1 \sim (\text{Mach number})^2$$

$$\text{Mach number} = u_1/v_{s,T}$$

HD phases of SNR evolution

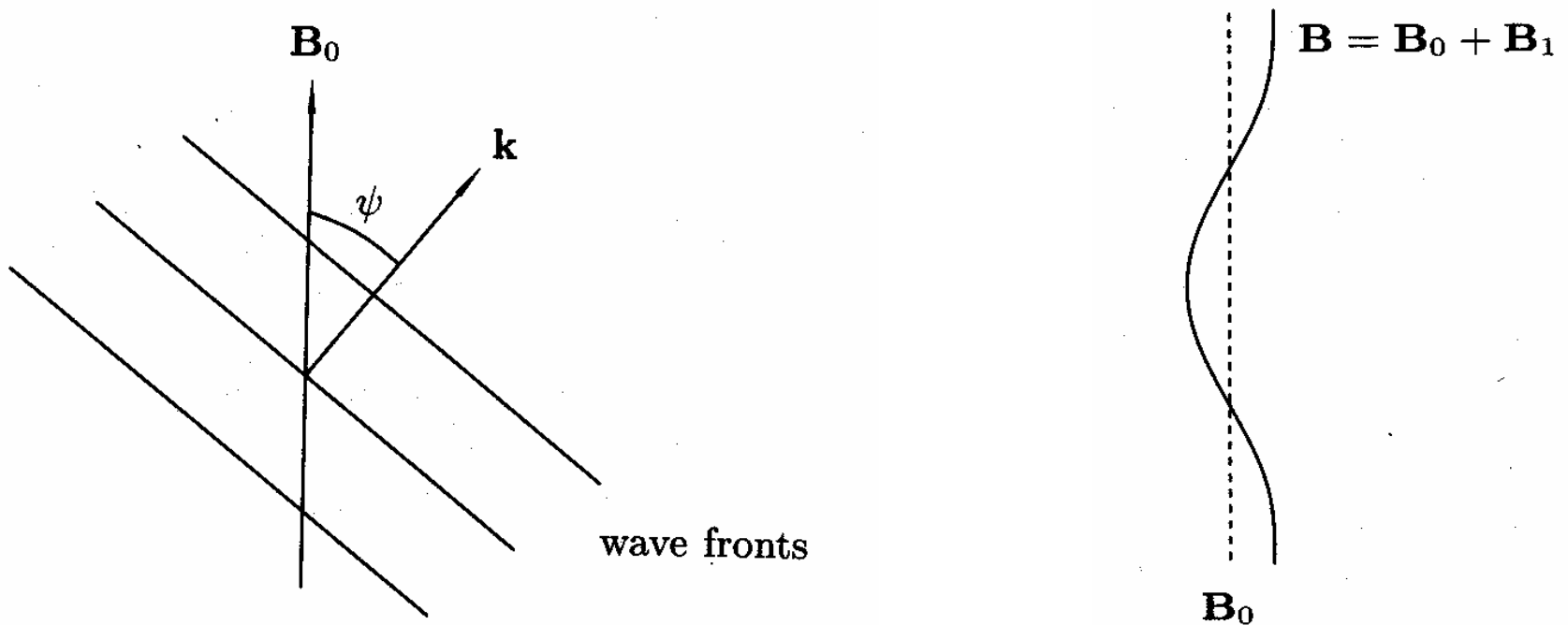
- First phase – free expansion phase ($M_s < M_e$),
till $3/4E_k \rightarrow U$ ($M_s \approx 3M_e$),
(for $1/2E_k \rightarrow U$, $M_s \approx M_e$).
- Second phase – adiabatic phase ($M_s \gg M_e$)
till $1/2E_k \rightarrow$ radiation
- Third phase – isothermal phase – formation
of thick shell
- Forth phase – dissipation into ISM





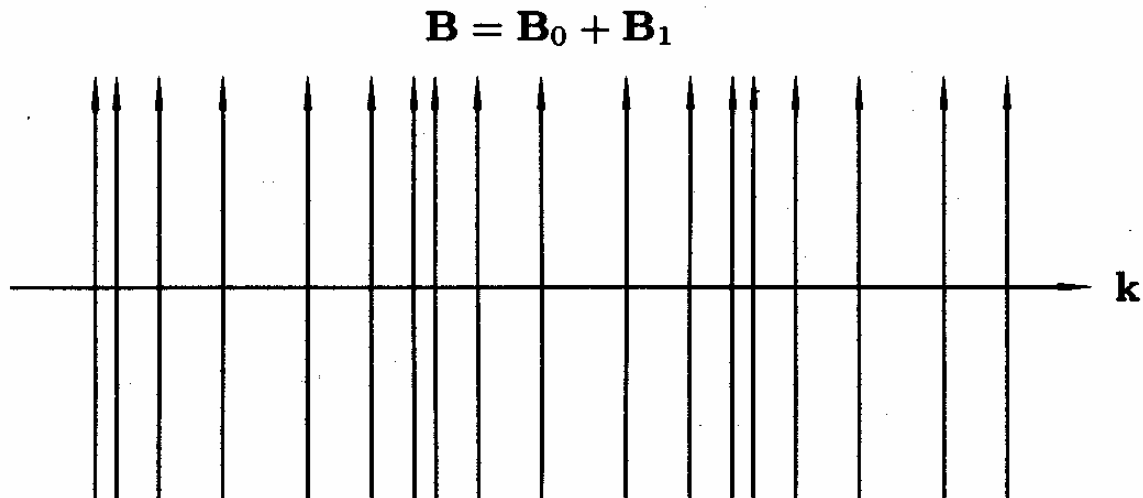
SNRs – MHD

- MHD
- Alfvén waves – transverse modes



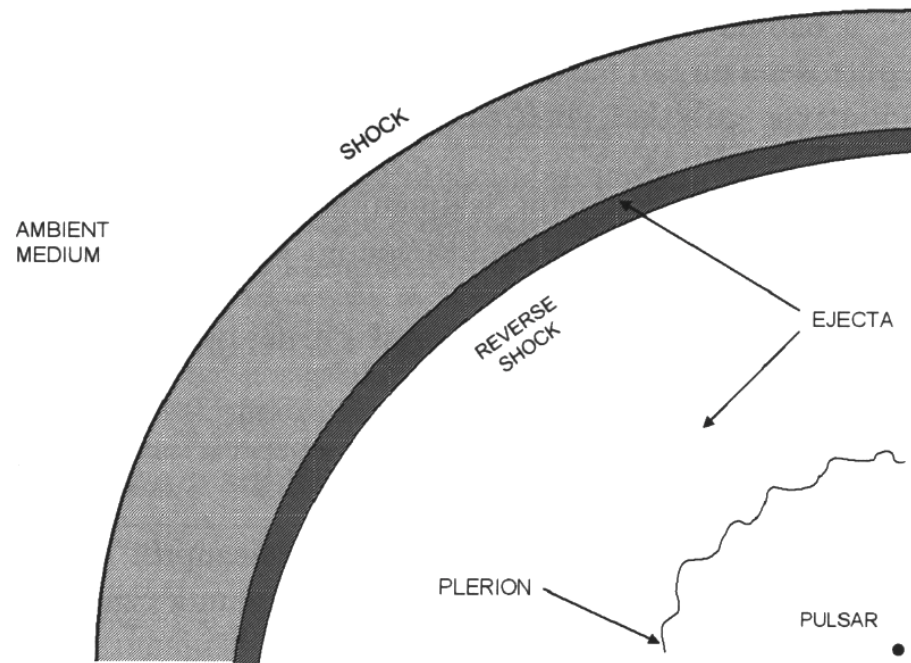
SNRs – MHD

- fast and slow magnetoacoustic waves
- longitudinal modes



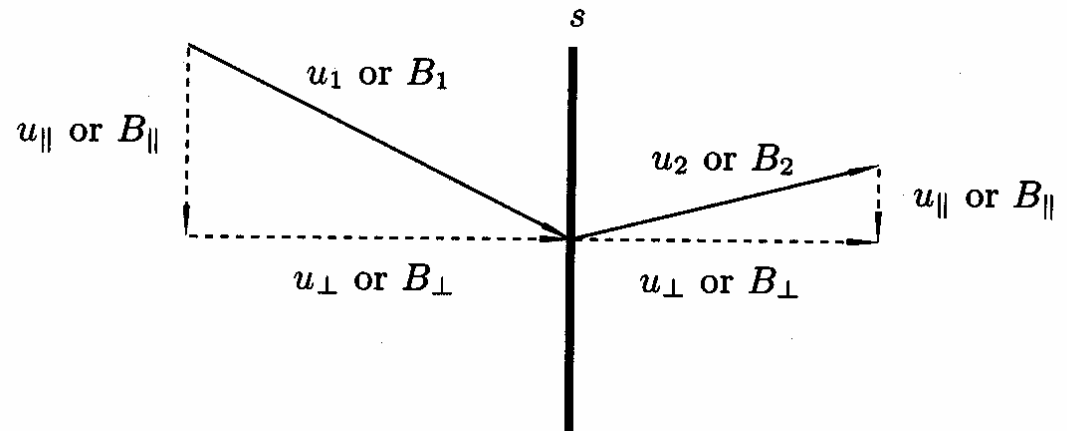
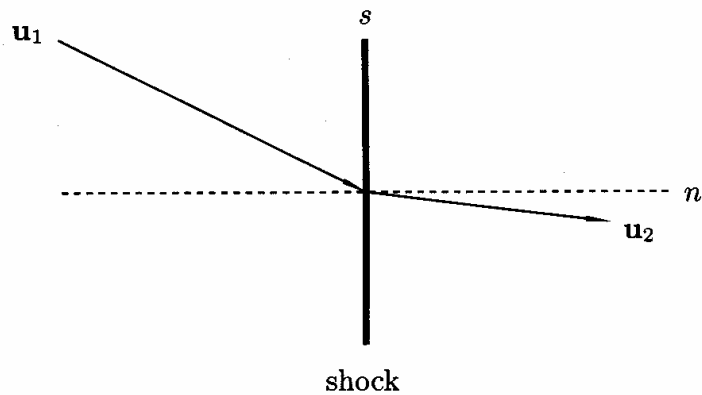
SNRs – MHD

- entropy waves – contact discontinuities
- discontinuities without flow



SNRs – MHD

- shock waves in the magnetized medium

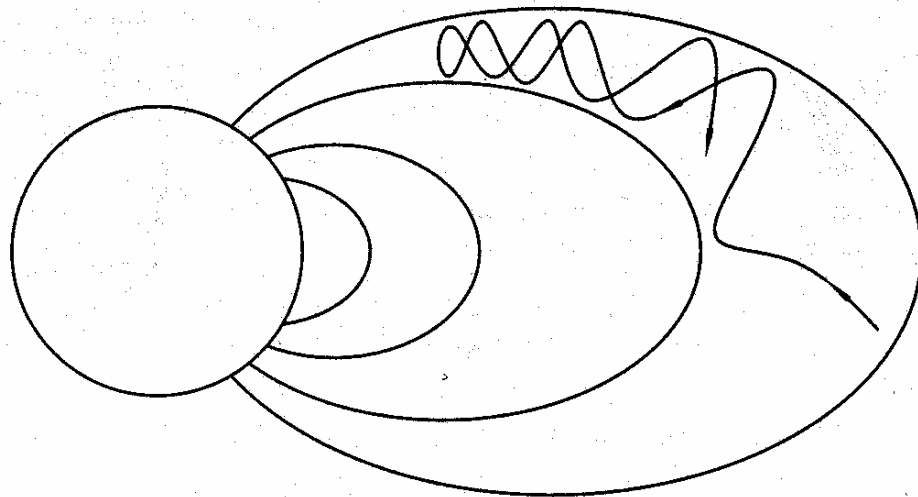


SNRs – particle acceleration

- transverse adiabatic invariant

$$p_{\perp}^2 / B = \text{const.}$$

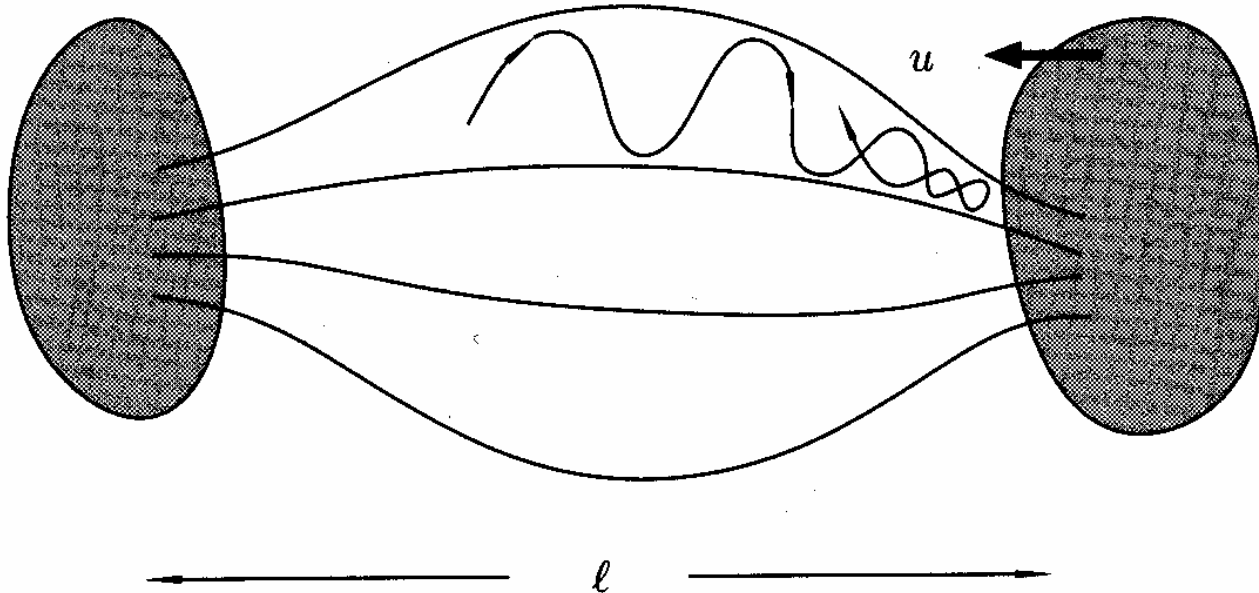
$$p_{\perp}^2 + p_{\parallel}^2 = \text{const.}$$



SNRs – particle acceleration

- longitudinal adiabatic invariant

$$p_{\parallel} l = \text{const.}$$



SNRs – particle acceleration

PHYSICAL REVIEW

VOLUME 75, NUMBER 8

APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI

Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

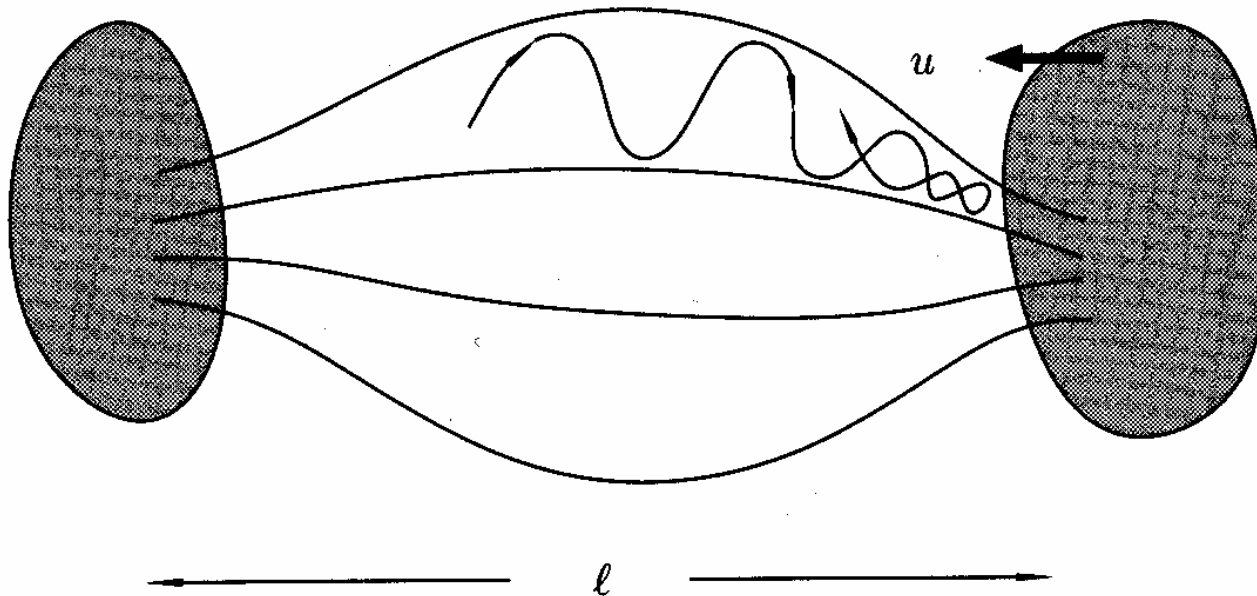
I. INTRODUCTION

IN recent discussions on the origin of the cosmic radiation E. Teller¹ has advocated the view that cosmic rays are of solar origin and are kept relatively near the sun by the action of magnetic fields. These views are amplified by Alfvén, Richtmyer, and Teller.² The argument against the conventional view that cosmic radiation may extend at least to all the galactic space is the very large amount of energy that should be present in form of cosmic radiation if it were to extend to such a huge space. Indeed, if this were the case, the mechanism of acceleration of the cosmic radiation should be extremely efficient.

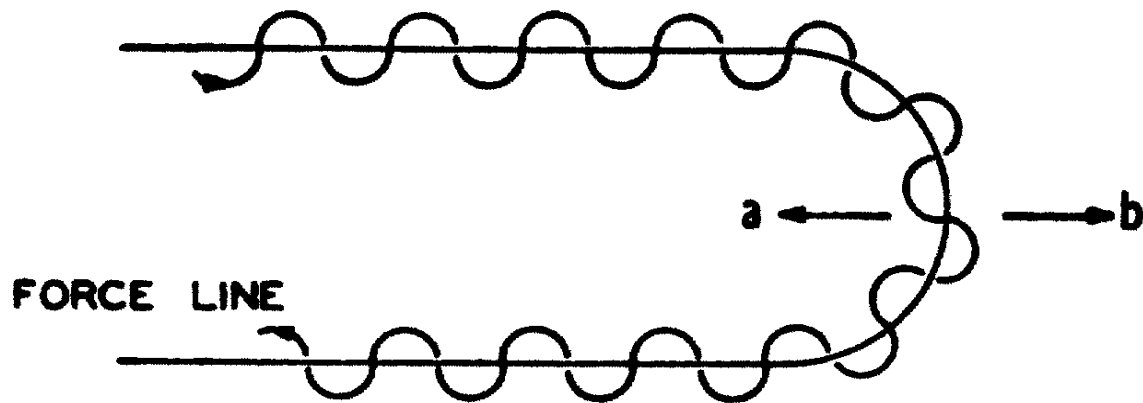
where H is the intensity of the magnetic field and ρ is the density of the interstellar matter.

One finds according to the present theory that a particle that is projected into the interstellar medium with energy above a certain injection threshold gains energy by collisions against the moving irregularities of the interstellar magnetic field. The rate of gain is very slow but appears capable of building up the energy to the maximum values observed. Indeed one finds quite naturally an inverse power law for the energy spectrum of the protons. The experimentally observed exponent of this law appears to be well within the range of the possibilities.

- Fermi acceleration (“Type A” in Fermi (1949))



- Fermi acceleration (“Type B” in Fermi (1949)) – affirmed in this paper

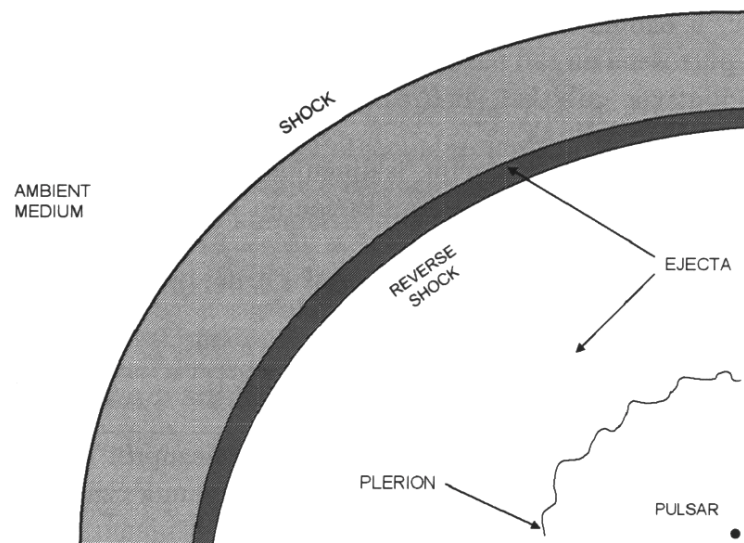


- In both cases

$$\Delta E / E \sim (v / c)^2$$

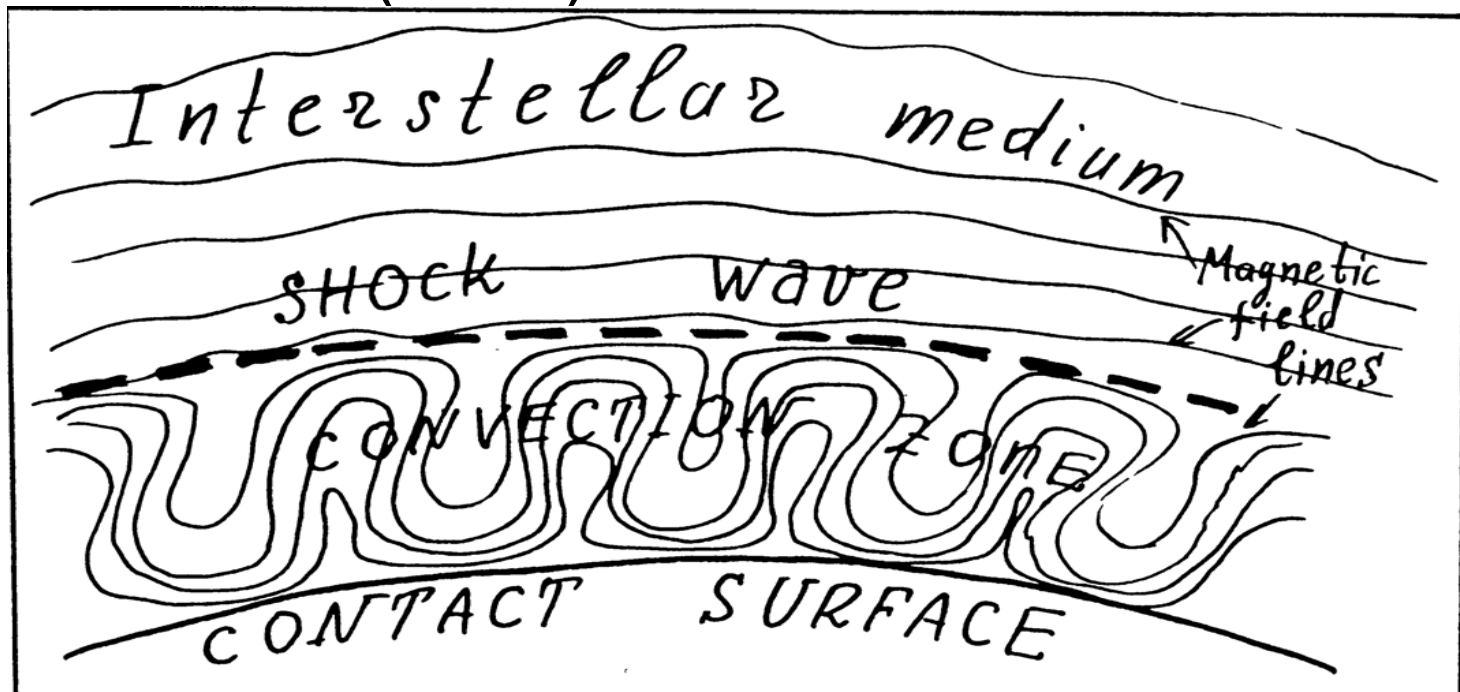
SNRs – particle acceleration

- diffuse shock acceleration – first order Fermi acceleration
- Bell (1978a,b), Blanford & Ostriker (1978, 1980), Drury (1983a,b), Malkov & Drury (2001)



SNRs – particle acceleration

- second order Fermi acceleration – turbulences in downstream region
- Scott & Chevalier (1975), Galinsky & Shevchenko (2007)



SNRs – particle acceleration

- DSA – diffusion only in pinch angle
- second order acceleration – diffusion in velocities
- hints for future research – modeling of both processes in the same time

**THANK YOU
VERY MUCH
FOR YOUR
ATTENTION!!!**