On a Simple Model of Nonlocal de Sitter Gravity

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(joint work with I. Dimitrijević, B. Dragovich, and J. Stanković)

SEMINAR OF DEPARTMENT OF ASTRONOMY

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- GTR (or ETG) assumes that Universe is four dimensional homogeneous and isotropic pseudo-Riemannian manifold M with metric $(g_{\mu\nu})$ of signature $(1,3)$.

- There exist three types of homogeneous and isotropic simple connected spaces of dimension 3:
  - sphere S^3 (of constant positive sectional curvature),
  - flat space $\mathbb{R}^3$ (of curvature equal 0),
  - hyperbolic space $\mathbb{H}^3$ (of constant negative sectional curvature).

- Generic metric in these spaces is of the form (Friedmann-Robertson-Walker metric (FRW)):

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right), \quad k \in \{-1, 0, 1\}, \quad (1)$$

where $a(t)$ is a cosmic scale factor which describes the evolution (in time) of Universe and parameter k which describes the curvature of the space.

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$$S = \int \left(\frac{R - 2\Lambda}{16\pi G c^4} + \mathcal{L}_m \right) \sqrt{-g}\, d^4x$$

where R is scalar curvature, $g = \det(g_{\mu\nu})$ is determinant of metric tensor, Λ is cosmological constant and $\mathcal{L}_m$ is Lagrangian of matter.

- The variation of the action S we obtain equations of motion:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad c = 1 \tag{2}$$

where $T_{\mu\nu}$ is the energy momentum tensor, $g_{\mu\nu}$ is metric tensor, $R_{\mu\nu}$ is Ricci tensor and R is scalar curvature.

- The energy momentum tensor for ideal fluid (matter in cosmology) is

$$T = \mathrm{diag}(-\rho g_{00}, g_{11} p, g_{22} p, g_{33} p), \tag{3}$$

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- Now, Einstein equation implies Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \qquad H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}.$$

- Hubble parameter describes the expansion of the Universe

$$H = \frac{\dot{a}}{a}. \tag{4}$$

- Despite to the great success of GRT, observational discoveries of 20th century imply that they could not be explained by GTR without additional matter.
- Problem of Bing Bang singularity.
- It means that GRT should be modified. There are two approaches:

(A1) Dark matter and energy

(A2) Modification of GTR, i.e. modification of its Lagrangian $\mathcal{L}$

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Dark matter and energy

- Dark matter is responsible for orbital speeds in galaxies, and dark energy is responsible for accelerated expansion of the Universe.
- If Einstein theory of gravity can be applied to the whole Universe then the Universe contains about 5% of ordinary matter, 27% of dark matter and 68% of dark energy.
- It means that 95% of total matter, or energy, represents dark side of the Universe, which nature is unknown.

Motivation for modification of Einstein theory of gravity

- The validity of General Relativity on cosmological scale is not confirmed.
- Dark matter and dark energy are not yet detected in the laboratory experiments.

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Different approaches to modification of Einstein theory of gravity

- Einstein General Theory of Relativity

From action

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using variational methods we get field equations

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$$R \longrightarrow f(R, \Lambda, R_{\mu\nu}, R^{\alpha}_{\mu\beta\nu}, \Box, \dots), \quad \Box = \nabla_\mu \nabla^\mu = \frac{1}{\sqrt{-g}}\partial_\mu \sqrt{-g}\, g^{\mu\nu}\, \partial_\nu$$

Gauss-Bonnet invariant

$$\mathcal{G} = R^2 - 4\, R^{\mu\nu}\, R_{\mu\nu} + R^{\alpha\beta\mu\nu}\, R_{\alpha\beta\mu\nu}$$

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- Under nonlocal modification of gravity we understand replacement of the scalar curvature R in the Einstein-Hilbert action by a suitable function $F(R, \Box)$, where $\Box = \nabla_\mu \nabla^\mu$ is d'Alembert operator and ∇_μ denotes the covariant derivative
- Let M be a four-dimensional pseudo-Riemannian manifold with metric $(g_{\mu\nu})$ of signature (1,3). We consider a class of nonlocal gravity models without matter, given by the following action

$$S = \int_M \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{H}(R)\,\mathcal{F}(\Box)\,\mathcal{G}(R) \right) \sqrt{-g}\, \mathrm{d}^4 x,$$

where $\mathcal{F}(\Box) = \sum_{n=0}^{\infty} f_n \Box^n$ is an analytic function of $\Box$, and Λ is cosmological constant.

- In the case of FRW metric the scalar curvature and d'Alambert operator are given by

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}, \quad \Box R = -\ddot{R} - 3\,H\,\dot{R}, \quad H = \frac{\dot{a}}{a}.$$

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- The action S_0 is Einstein-Hilbert action without matter its variation is

$$\delta S_0 = \int_M G_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\, d^4x + \Lambda \int_M g_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}\, d^4x, \tag{5}$$

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where

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$$S=\int_M \Big(\frac{R-2\Lambda}{16\pi G}+\mathcal{H}(R)\,\mathcal{F}(\Box)\,\mathcal{G}(R)\Big)\sqrt{-g}\,d^4x,$$

for the following cases:

1. $\mathcal{H}(R)=R$, $\mathcal{G}(R)=R$,
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1. model $\mathcal{H}(R) = R$, $\mathcal{G}(R) = R$.

- Using ansatz $\Box R = r R + s$ we found three types of non-singular bounced solutions for the scalar factor $a(t) = a_0(\sigma e^{\lambda t} + \tau e^{-\lambda t})$.
- Solutions exist for all three values of parameter $k = 0, \pm 1$, under certain conditions on function $\mathcal{F}(\Box)$, and parameters $\sigma, \tau, \lambda, \Lambda, k$.
- Obtained results generalize known cases in literature: in the first case $a(t) = a_0 \cosh\left(\sqrt{\frac{\Lambda}{3}} t\right)$, in the second and third case for $k = 0$ we obtain de Sitter solution.
- All obtained solutions satisfy $\ddot{a}(t) = \lambda^2 a(t) > 0$, i.e. are consistent with observational data.

2. model $\mathcal{H}(R) = R^{-1}$, $\mathcal{G}(R) = R$.

- Non-locality, $R^{-1}\mathcal{F}(\Box)R$, is invariant to transformation $R \longrightarrow cR$, $c \in \mathbb{R}^*$.
- there are cosmological solutions of form $a(t) = a_0|t - t_0|^\alpha$, in the case $k = 0$, for $\alpha \neq 0, 1/2$ and $3\alpha \in 1 + 2\mathbb{N}$, in cases $k \neq 0$, for $\alpha = 1$.
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- We considered case with scale factor in the form $a(t) = a_0 \exp(-\frac{\gamma}{12}t^2)$
- For $p = q = 1$ there are infinite number of solutions, and constants γ and Λ satisfy $\gamma = -12\Lambda$.
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- The action (M4) is **limit case** od the action (MS) since: the expansion of $\sqrt{R - 2\Lambda} = \sqrt{-2\Lambda}\sqrt{1 - \frac{R}{2\Lambda}}$ where $|R| \ll |2L|$.

- Linear approximation in $R/2\Lambda$ gives $\sqrt{R - 2\Lambda} = \sqrt{-2\Lambda}\,(1 - \frac{R}{4\Lambda})$, then the nonlocal term in (MS) becomes

$$\sqrt{R - 2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R - 2\Lambda} \simeq -\frac{R}{8\Lambda}\,(R - 4\Lambda)\,\mathcal{F}(\Box)\,(R - 4\Lambda),$$

- Recently, we have considered classes of nonlocal gravity models with cosmological constant Λ and without matter, given by

$$\text{(M4)} \qquad S = \frac{1}{16\pi G}\int_M \big(R - 2\Lambda + (R - 4\Lambda)\,\mathcal{F}(\Box)(R - 4\Lambda)\big)\sqrt{-g}\;\mathrm{d}^4x,$$

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- Let us consider model (MS) in more details, so we

$$S = \frac{1}{16\pi G} \int_M \left(R - 2\Lambda + \sqrt{R - 2\Lambda}\,\mathcal{F}(\Box)\sqrt{R - 2\Lambda}\right)\sqrt{-g}\,d^4x, \qquad (9)$$

where $\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} f_n \Box^n + \sum_{n=1}^{+\infty} f_{-n} \Box^{-n}$

- It is a ▸ very special case since the EOM (8), for $\mathcal{G}(R) = \sqrt{R - 2\Lambda}$, is simplified to

$$(G_{\mu\nu} + \Lambda g_{\mu\nu})(1 + \mathcal{F}(q)) + \frac{1}{2}\mathcal{F}'(q) S_{\mu\nu}(\sqrt{R - 2\Lambda}, \sqrt{R - 2\Lambda}) = 0, \quad (10)$$

where we take $q = \zeta\Lambda$.

- It is evident that EOM (10) are satisfied if $\mathcal{F}(q) = -1$ and $\mathcal{F}'(q) = 0$.
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$$\mathcal{F}(\Box) = 1 + \sum_{n=1}^{+\infty} \tilde{f}_n \left[\left(\frac{\Box}{q}\right)^n + \left(\frac{q}{\Box}\right)^n\right] = 1 - \frac{1}{2e}\left(\frac{\Box}{q} e^{\frac{\Box}{q}} + \frac{q}{\Box} e^{\frac{q}{\Box}}\right), \quad q \neq 0.$$

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1. **Cosmological solution in the flat Universe** $(k = 0)$

1.1. Solutions of the form $a(t) = A\, t^{\alpha} e^{\gamma t^2}$

- There are two solutions:

1.2. New solutions of the form $a(t) = (\alpha e^{\lambda t} + \beta e^{-\lambda t})^{\gamma}$

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$$\gamma = \frac{2}{3}, \qquad q = \frac{3}{8}\Lambda, \qquad \lambda = \pm\sqrt{\frac{3}{8}\Lambda}.$$

- When $\alpha\beta \neq 0$, we have the following two special solutions:

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1. **Cosmological solution in the flat Universe** $(k = 0)$

- For $\alpha = 0$ or $\beta = 0$, we have also two cosmological solutions with $\gamma = \frac{2}{3}$:

$$a_7(t) = A \sin^{\frac{2}{3}} \Big(\sqrt{-\frac{3}{8}\Lambda}\, t\Big), \qquad \mathcal{F}\Big(\frac{3}{8}\Lambda\Big) = -1, \ \ \mathcal{F}'\Big(\frac{3}{8}\Lambda\Big) = 0,$$

$$a_8(t) = A \cos^{\frac{2}{3}} \Big(\sqrt{-\frac{3}{8}\Lambda}\, t\Big), \qquad \mathcal{F}\Big(\frac{3}{8}\Lambda\Big) = -1, \ \ \mathcal{F}'\Big(\frac{3}{8}\Lambda\Big) = 0.$$

1. **Cosmological solution in the flat Universe** $(k = 0)$

1.3. New solutions of the form $a(t) = (\alpha \ \sin \lambda t + \beta \ \cos \lambda t)^{\gamma}$

- For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ, $\gamma = \frac{2}{3}$. Taking $\beta = \pm\alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_5(t) = A\left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda\, t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0,$$

$$a_6(t) = A\left(1 - \sin\left(2\sqrt{-\frac{3}{8}\Lambda\, t}\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\left(\frac{3}{8}\Lambda\right) = -1, \ \ \mathcal{F}'\left(\frac{3}{8}\Lambda\right) = 0.$$

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   1.3. New solutions of the form $a(t) = (\alpha \; \sin \lambda t + \beta \; \cos \lambda t)^{\gamma}$

- For $\alpha \neq 0$ and $\beta \neq 0$ there are only possibility for γ, $\gamma = \frac{2}{3}$. Taking $\beta = \pm\alpha$, and $A = \alpha^{\frac{2}{3}}$, we have the following two solutions:

$$a_5(t) = A\left(1 + \sin\left(2\sqrt{-\frac{3}{8}\Lambda}\, t\right)\right)^{\frac{1}{3}}, \qquad \mathcal{F}\big(\frac{3}{8}\Lambda\big) = -1, \;\; \mathcal{F}'\big(\frac{3}{8}\Lambda\big) = 0,$$

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2. **Cosmological solution in the open and closed Universe** $(k = \mp 1)$

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- The corresponding Hubble parameter, acceleration and the scalar curvature are:

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- The expressions (11) implies that $\bar{w}(t) \to -1$ when $t \to \infty$, what corresponds to an analog of Λ dark energy dominance in the standard cosmological model.
- It means that this nonlocal gravity model with cosmological solution $a(t) = A\, t^{\frac{2}{3}}\, e^{\frac{\Lambda}{14} t^2}$ describes some effects usually attributed to the dark matter and dark energy.
- This solution is invariant under transformation $t \to -t$ and singular at cosmic time $t = 0$.
- Let us recall, the second Friedman equation

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \qquad (12)$$

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- Then we can rewrite the previous equation as,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho_r + \frac{8\pi G}{3}\rho_m - \frac{k}{a^2} + \frac{\Lambda}{3}$$
$$= \frac{8\,C_r\pi G}{a^4} + \frac{8\,C_m\pi G}{a^3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- It follows

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

- Observational data obtained by Planck-2018 for the ΛCDM model:

  $t_0 = (13.801 \pm 0.024) \times 10^9$yr – age of the universe,
  $H(t_0) = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,
  $\Omega_m = 0.315 \pm 0.007$– matter density parameter,
  $\Omega_\Lambda = 0.685$ – Λ density parameter,
  $w_0 = -1.03 \pm 0.03$– ratio of pressure to energy density.

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- Observational data obtained by Planck-2018 for the ΛCDM model:

$t_0 = (13.801 \pm 0.024) \times 10^9$yr – age of the universe,

$H(t_0) = (67.40 \pm 0.50)$ km/s/Mpc – Hubble parameter,

$\Omega_m = 0.315 \pm 0.007$– matter density parameter,

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$$H_1(t) = \frac{\dot{a}_1}{a_1} = \frac{2}{3}\frac{1}{t} + \frac{1}{7}\Lambda t,$$

taking $H_1(t_0) = H(t_0)$ we calculate $\Lambda_1 = 1.05 \times 10^{-35}\mathrm{s}^{-2}$ that differs from $\Lambda = 3H^2(t_0)\,\Omega_\Lambda = 0.98 \times 10^{-35}\mathrm{s}^{-2}$ (by ΛCDM model).

- We also computed

$$\ddot{a}_1(t_0)/a_1(t_0) = 2.7 \times 10^{-36} s^{-2}$$
$$R(t_0) = 4.5 \times 10^{-35} s^{-2} \quad \text{and consequently}$$
$$R(t_0) - 2\Lambda = 2.4 \times 10^{-35} s^{-2}.$$

- Replacing solution $a_1(t)$ with $k = 0$, Friedman equations give

$$\bar{\rho}_1(t) = \frac{3}{8\pi G}\Big(H_1^2(t) - \frac{\Lambda_1}{3}\Big) = \frac{3}{8\pi G}\Big(\frac{4}{9}t^{-2} - \frac{1}{7}\Lambda_1 + \frac{1}{49}\Lambda_1^2 t^2\Big),$$
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- For $t = t_0$, from previous formula, and from ΛCDM model we have

$$\bar{\rho}_1(t_0) = 2.26 \times 10^{-30} \frac{g}{cm^3},$$

$$\rho(t_0) = \frac{3}{8\pi G}\left(H_0^2 - \frac{\Lambda}{3}\right) = 2.68 \times 10^{-30} \frac{g}{cm^3}.$$

- Then, for vacuum energy density of background solution $a_1(t)$ and ΛCDM model, we have

$$\rho(t_0) - \bar{\rho}_1(t_0) = \frac{\Lambda_1 - \Lambda}{8\pi G} = \rho_{\Lambda_1} - \rho_{\Lambda} = 0.42 \times 10^{-30} \frac{g}{cm^3},$$

- Critical energy density: $\rho_c = \dfrac{3\,H_0^2}{8\,\pi\,G} = 8.51 \times 10^{-30} \dfrac{g}{cm^3}$
- and consequently,

$$\Omega_{\Lambda_1} = \frac{\rho_{\Lambda_1}}{\rho_c} = 0.734, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_c} = 0.685, \quad \Delta\Omega_{\Lambda} = \Omega_{\Lambda_1} - \Omega_{\Lambda} = 0.049, \qquad (13)$$

$$\Omega_{m_1} = \frac{\bar{\rho}_1(t_0)}{\rho_c} = 0.266,\ \Omega_m = \frac{\rho(t_0)}{\rho_c} = 0.315,\ \Delta\Omega_m = \Omega_m - \Omega_{m_1} = 0.049. \ (14)$$

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- According to (13) and (14), we obtain that $\Omega_{m_1} = 26,6\%$ corresponds to dark matter and $\Delta\Omega_m = \Delta\Omega_\Lambda = 4.9\%$ is related to visible matter, what is in a very good agreement with the standard model of cosmology.
- Efective presure. At the beginning, $\bar{p}_1(0) = \frac{\Lambda_1}{56\pi G} > 0$, then decreases and equals zero at $t = \sqrt{\frac{7}{3\Lambda_1}} = 4.71 \times 10^{17}\, s = 14,917 \times 10^9 yr$.
- ▸ According to (11), we have parameter $\bar{w}_1(t) = \frac{\bar{p}_1(t)}{\bar{\rho}_1(t)}$ which has future behavior in agreement with standard model of cosmology, i.e. $\bar{w}_1(t \to \infty) \to -1$.
- Note that ▸ the Hubble parameter has minimum at $t_{min} = 21.1 \times 10^9 yr$ and it is $H_1(t_{min}) = 61.72 km/s/Mpc$. It also, follows that the Universe experiences decelerated expansion during matter dominance, that turns to acceleration at time $t_{acc} = 7.84 \times 10^9 yr$ when, $\ddot{a} = 0$.

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- We want to investigate our model outside the spherically symmetric massive body - it is natural to consider a generalization of the Schwarz-schild-de Sitter (SdS) metric starting from the standard Schwarzschild expression,

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2. \qquad (15)$$

- The corresponding scalar curvature R of above metric (15)

$$R = \frac{2}{r^2} - \frac{2}{r^2 B(r)} - \frac{2A'(r)}{rA(r)B(r)} + \frac{A'(r)^2}{2A(r)^2 B(r)} + \frac{2B'(r)}{rB^2(r)} + \frac{A'(r)B'(r)}{2A(r)B(r)^2} - \frac{A''(r)}{A(r)B(r)} \qquad (16)$$

- We need to solve the equation

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SdS metric-GC

- The corresponding scalar curvature R of above metric (15)

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- It makes sense to suppose that solution of equation (17) is of the form

$$A(r) = A_0(r) - \alpha(r), \quad B(r) = \frac{1}{A_0(r) - \beta(r)}, \tag{20}$$

where $\alpha(r)$ and $\beta(r)$ are some dimensionless functions. When $\zeta = q/\Lambda \to 0$, then nonlocal operator nonlocal de Sitter $\sqrt{dS}$ gravity model (9) becomes local.
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R = {} & \frac{2}{r^2}(1 - A_0 + \beta) + 2\frac{A_0 - \beta}{A_0 - \alpha}(A_0' - \alpha')\Big(\frac{1}{4}\frac{A_0' - \alpha'}{A_0 - \alpha} - \frac{1}{r}\Big) \\
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- If we substitute expressions (21) and (22) in eigenvalue problem for $\Box$ operator, we will get a differential equation in $\alpha(r)$ and $\beta(r)$. Since, in the local case holds $B_0(r) = 1/A_0(r)$, there is a sense to take $B(r) = 1/A(r)$ in the nonlocal case, it means $\beta(r) = \alpha(r)$, and it yields:

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R(r) = \frac{1}{r^2}\big[2 - 2A(r) - 4rA'(r) - r^2A''(r)\big] = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}\big[r^2(1 - A(r))\big], \qquad (23)
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- One gets an ordinary nonlinear differential equation of the fourth order, since it is nonlinear, it is a very difficult task to find the corresponding exact solution. In the sequel of this lecture we will turn our attention to the corresponding linear differential equation: it means we will limit ourselves to studying the Schwarzshild-de Sitter metric in weak gravity field approximation.

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- It is like considering gravity field far from a massive body, so $\Box$ can be replaced by the Laplacian $\triangle$ in equation (26). In such case we will take $A(r) \approx 1$ in (26), that is

$$A(r) = A_0(r) - \alpha(r) = 1 - \frac{\mu}{r} - \frac{\Lambda r^2}{3} - \alpha(r) \approx 1, \tag{28}$$

i.e. if the following is satisfied,

$$\frac{\mu}{r} \ll 1, \qquad \frac{\Lambda r^2}{3} \ll 1, \qquad |\alpha(r)| \ll 1. \tag{29}$$

- Applying approximation (28) in (26), we get the following simple equation linear in $u(r)$:

$$\triangle u = qu, \quad \text{that is} \quad \frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} = qu, \quad u = \sqrt{R - 2\Lambda}. \tag{30}$$

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- After the linearization of $\sqrt{R-2\Lambda}$, we get

$$(R-4\Lambda)'' + \frac{2}{r}(R-4\Lambda)' = q(R-4\Lambda), \tag{31}$$

and using (25) we obtain the following linear differential equation,

$$\alpha'''' + \frac{6}{r}\alpha''' + \frac{2}{r^2}\alpha'' - \frac{4}{r^3}\alpha' + \frac{4}{r^4}\alpha = q(\alpha'' + \frac{4}{r}\alpha' + \frac{2}{r^2}\alpha). \tag{32}$$

- Previous equation (32) has a general solution for $q = \zeta\Lambda$,

$$\alpha(r) = \frac{C_1}{r} + \frac{C_2}{r^2} + C_3 e^{-\sqrt{q}r}\left(\frac{1}{qr} + \frac{2}{q^{\frac{3}{2}} r^2}\right) + C_4 e^{\sqrt{q}r}\left(\frac{1}{qr} - \frac{2}{q^{\frac{3}{2}} r^2}\right). \tag{33}$$

- There are four constants ($C_1 - C_4$) and we want to chose them such that the appropriate particular solution for $\alpha(r)$ has some physical meaning, i.e. $\alpha(r) \to 0$ when $\zeta \to 0$.

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- One such choice is: $C_1 = -\delta/\sqrt{q}$, $C_2 = 2\delta/q$, $C_3 = -\delta\sqrt{q}$, $C_4 = 0$.
- C_4 has to vanish, since we have exclude term with $e^{\sqrt{q}r}$ in (33).
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$$\alpha(r) = -\frac{\delta}{\sqrt{q}r}\left(1 + e^{-\sqrt{q}r}\right) + \frac{2\delta}{qr^2}\left(1 - e^{-\sqrt{q}r}\right), \quad q = \zeta\Lambda, \tag{34}$$

  where δ is dimensionless parameter.
- Since integration constants C_1, C_2, C_3 are proportional to δ, and $C_4 = 0$, we reduced the number of parameters from 4 to 1. We have two free parameters (δ and ζ) which should be determined from measurements.
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The Rotation Curves of Spiral Galaxies

- The rotation curves of spiral galaxies play an important role, since we need them to determine the amount and distribution of dark matter comparing to visible matter.
- We want examine whether the $\sqrt{dS}$ gravitational model gives the possibility of describing the rotation curves of spiral galaxies.

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- The rotation curves of spiral galaxies play an important role, since we need them to determine the amount and distribution of dark matter comparing to visible matter.
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- We start with $A(r)$ given by (36) and present the corresponding gravitational potential $\Phi(r)$, which is

$$\Phi(r) = \frac{c^2}{2}(1 - A(r)) = \frac{GM}{r} + \frac{\Lambda c^2 r^2}{6} + \frac{c^2}{2}\alpha(r). \qquad (37)$$

- The corresponding gravitational acceleration for potential (37) is

$$a_g(r) = -\frac{\partial \Phi}{\partial r}$$

$$= \frac{GM}{r^2} - \frac{\Lambda c^2 r}{3} + \frac{\delta c^2}{\sqrt{q} r^2}\left(\frac{2}{\sqrt{q} r} - \frac{1}{2}\right) - \frac{\delta c^2}{r}\left(\frac{1}{2} + \frac{3}{2\sqrt{q} r} + \frac{2}{q r^2}\right) e^{-\sqrt{q}\, r}.$$

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$$\begin{aligned} a_g(r) &= -\frac{\partial \Phi}{\partial r} \\ &= \frac{GM}{r^2} - \frac{\Lambda c^2 r}{3} + \frac{\delta c^2}{\sqrt{q} r^2}\left(\frac{2}{\sqrt{q} r} - \frac{1}{2}\right) - \frac{\delta c^2}{r}\left(\frac{1}{2} + \frac{3}{2\sqrt{q} r} + \frac{2}{q r^2}\right) e^{-\sqrt{q}\, r}. \end{aligned}$$

- Velocity of the rotation curve $\bar{v}(r)$ is

$$\begin{aligned} \bar{v}(r) &= \sqrt{a_g(r)\, r} \qquad (38) \\ &= c\sqrt{\frac{GM}{c^2 r} - \frac{\Lambda r^2}{3} + \frac{\delta}{\sqrt{q} r}\left(\frac{2}{\sqrt{q} r} - \frac{1}{2}\right) - \delta\left(\frac{1}{2} + \frac{3}{2\sqrt{q} r} + \frac{2}{q r^2}\right) e^{-\sqrt{q}\, r}}. \end{aligned}$$

- We checked the validity of the obtained formula for the circular velocity (38) on two cases: the Milky Way galaxy and the spiral galaxy M33.
- The values of parameters δ and ζ in (38) are estimated by best fitting of measured data using the least-squares method.
- **Milky Way case.** The Milky Way rotation curve data have taken from recent paper Jiao, Y.; Hammer, F.; Wang, H.; Wang, J.; Amram, P.; Chemin, L.; Yang, Y., Detection of the Keplerian decline in the Milky Way rotation curve. *A&A* **2023**, *678*, A208.
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r [kpc]	v [km/s]	Δv [km/s]	$\bar{v}$ [km/s]	**relative error** δv[%]
9.5	221.75	3.17	217.36	1.98
10.5	223.32	3.02	220.19	1.40
11.5	220.72	3.47	221.93	0.55
12.5	222.92	3.19	222.72	0.09
13.5	224.16	3.48	222.66	0.67
14.5	221.60	4.20	221.85	0.11
15.5	218.79	4.75	220.37	0.72
16.5	216.38	4.96	218.28	0.88
17.5	213.48	6.13	215.63	1.01
18.5	209.17	4.42	212.47	1.58
19.5	206.25	4.63	208.83	1.25
20.5	202.54	4.40	204.77	1.10
21.5	197.56	4.62	200.29	1.38
22.5	197.00	3.81	195.42	0.80
23.5	191.62	12.95	190.17	0.75
24.5	187.12	8.06	184.57	1.36
25.5	181.44	19.58	178.62	1.55
26.5	175.68	24.68	172.32	1.91

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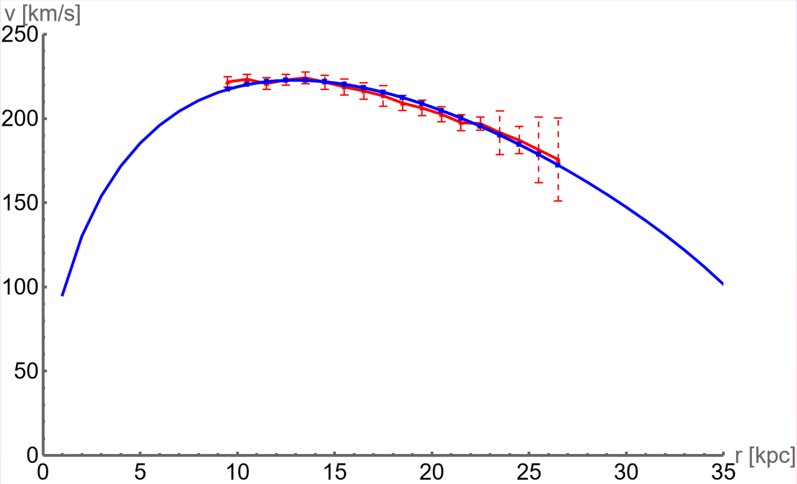
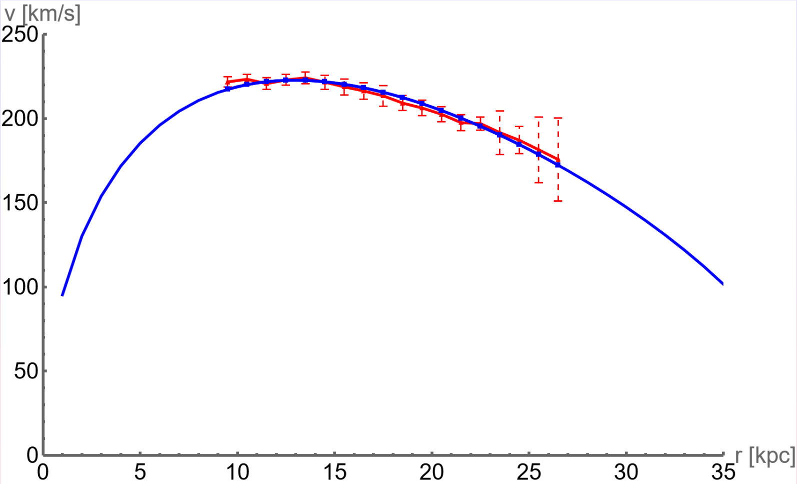


Figure: Rotation curve for the Milky Way galaxy. Red points are measured observational values from Table 1 and blue curve is computed $\bar{v}(r)$ by formula (38), where $\delta = 1.9 \times 10^{-5}$, $\zeta = 4.4 \times 10^{10}$, $\Lambda = 10^{-52}\mathrm{m}^{-2}$ and $M = 4.28 \times 10^{6} M_{\odot}$.



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r [kpc]	v [km/s]	Δv [km/s]	$\bar{v}$ [km/s]	relative error δv[%]
0.5	42.0	2.4	35.62	15.18
1.0	58.8	1.5	49.61	15.63
1.5	69.4	0.4	59.83	13.79
2.0	79.3	4.0	68.02	14.22
2.4	86.7	1.8	73.59	15.12
2.9	91.4	3.1	79.64	12.86
3.4	94.2	4.8	84.90	9.88
3.9	96.5	5.5	89.51	7.25
4.4	99.8	3.9	93.58	6.23
4.9	102.1	1.7	97.21	4.80
5.4	103.6	0.4	100.44	3.05
5.9	105.9	0.7	103.32	2.44
6.4	105.7	1.7	105.90	0.19

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6.8	106.8	2.2	107.76	0.90
7.3	107.3	3.0	109.86	2.39
7.8	108.3	4.0	111.73	3.17
8.3	109.7	4.0	113.34	3.37
8.8	112.0	4.8	114.86	2.5
9.3	116.1	2.2	116.15	0.045
9.8	117.2	2.5	117.27	0.06
10.3	116.5	6.5	118.24	1.49
10.8	115.7	8.1	119.07	2.91
11.2	117.4	8.2	119.63	1.9
11.7	116.8	8.9	120.22	2.93
12.2	115.7	9.6	120.69	4.31
12.7	115.1	7.7	121.05	5.17
13.2	117.1	5.1	121.30	3.58
13.7	118.2	3.2	121.45	2.75
14.2	118.4	1.4	121.50	2.62
14.7	118.2	1.8	121.47	2.76
15.1	117.5	2.4	121.38	3.30
15.6	119.6	0.8	121.19	1.33

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16.6	122.6	0.5	120.59	1.64
17.1	124.1	2.9	120.17	3.16
17.6	125.0	2.2	119.69	4.24
18.1	125.5	2.5	119.15	5.06
18.6	125.2	8.1	118.54	5.32
19.1	122.0	9.8	117.87	3.38
19.5	120.4	8.5	117.29	2.58
20.0	114.0	26.6	116.52	2.21
20.5	110.0	34.6	115.70	5.18
21.0	98.7	27.4	114.82	16.33
21.5	100.1	33.4	113.89	13.77
22.0	104.3	35.2	112.91	8.25
22.5	101.2	27.4	111.88	10.56
23.0	123.5	39.1	110.81	10.27
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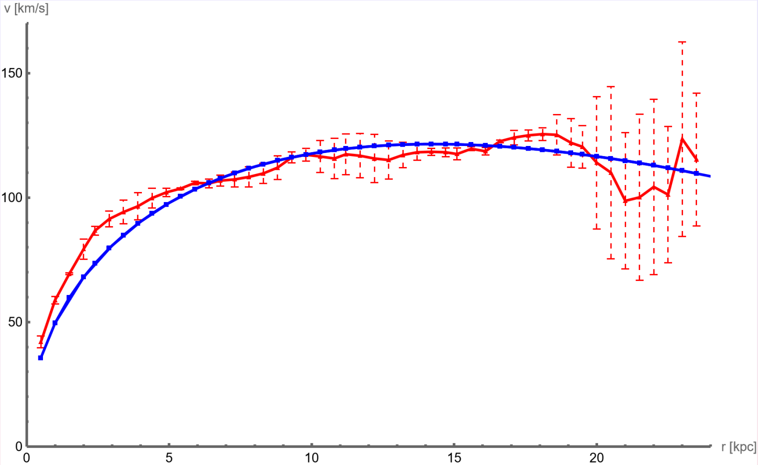
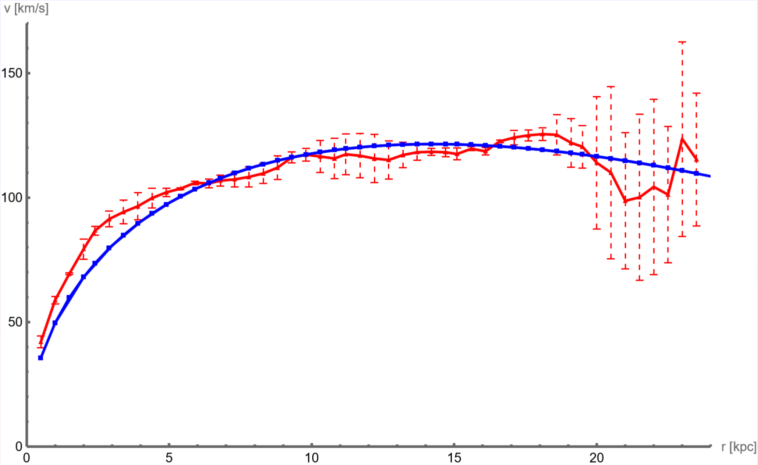


Figure: Rotation curve for spiral galaxy M33. Red points are measured observational values and blue line is computed $\bar{v}(r)$ by formula (38), where $\delta = 5.7 \times 10^{-6}$, $\zeta = 3.62 \times 10^{10}$, $\Lambda = 10^{-52}\text{m}^{-2}$ and $M = 1.5 \times 10^{3} M_{\odot}$.



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Concluding Remarks

- In our previously investigations of this model, we obtained results on the evolution of the universe, where the effects that are usually attributed to dark energy and dark matter can be described by the nonlocality of the gravity model $\sqrt{dS}$.
- Here, we found the Schwarzschild-de Sitter metric in the form of $A(r)$ (36), what corresponds to the weak gravity approximation and the linearization of nonlinear differential equation (26): a fourth-order linear differential equation for the Schwarzschild-de Sitter metric was obtained.
- A general solution linearized equation (32) was found. A particular solution of $\alpha(r)$ was found (34) such that it satisfies the necessary condition that it tends to zero when the nonlocality vanishes.

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- The obtained results were tested on the rotation curves of the Milky Way and the spiral galaxy M33: the rotation curves were observed in the domain: 9.5 –26.5 kpc for the Milky Way galaxy and 0.5 –23.5 kpc for the M33 galaxy.
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- Let us start with the action

$$S = \frac{1}{16\pi G}\int \sqrt{-g}\, R\, d^4x + \frac{1}{8\pi G}\int \sqrt{-g}\big(-\frac{1}{2}\,\nabla_\mu\varphi\nabla^\mu\varphi - V(\varphi)\big)d^4x. \tag{39}$$

- By variation of the previous action with respect to metric $g^{\mu\nu}$ we obtain

$$\frac{1}{16\pi G}G_{\mu\nu} + \frac{1}{8\pi G}\Big(\frac{1}{4}g_{\mu\nu}\nabla^\rho\varphi\nabla_\rho\varphi + \frac{1}{2}g_{\mu\nu}V(\varphi) - \frac{1}{2}\nabla_\mu\varphi\nabla_\nu\varphi\Big) = 0. \tag{40}$$

- Variation over φ yields $\Box\varphi = V'(\phi)$. The corresponding EOM are:

$$G_{\mu\nu} = 8\,\pi G\, T_{\mu\nu}, \qquad \Box\varphi = V'(\varphi). \tag{41}$$

- Now, we obtain

$$8\pi G\rho = \frac{1}{2}\dot\varphi^2 + V(\varphi), \qquad 8\pi G p = \frac{1}{2}\dot\varphi^2 - V(\varphi). \tag{42}$$

Finally, we have

$$8\pi G(\rho + p) = \dot\varphi^2 \qquad 4\pi G(\rho - p) = V(\varphi). \tag{43}$$

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- Corresponding effective density and pressure for this solution are:

$$\rho = \frac{2t^{-2} + \frac{9}{98}\Lambda^2 t^2 - \frac{9}{14}\Lambda}{12\pi G}, \qquad p = -\frac{\Lambda}{56\pi G}\left(\frac{3}{7}\Lambda t^2 - 1\right). \qquad (44)$$

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$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\left(R - 2\Lambda + \sqrt{R-2\Lambda}\,\mathcal{F}(\Box)\,\sqrt{R-2\Lambda}\right)$$
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THANK YOU FOR

YOUR ATTENTION !!!

Non-trivial Christoffel symbols of Friedman – Robertson – Walker metric

$$\Gamma^1_{01} = \frac{\dot a}{a} \qquad \Gamma^2_{02} = \frac{\dot a}{a} \qquad \Gamma^3_{03} = \frac{\dot a}{a}$$

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$$R_{0110} = \frac{a\,\ddot a}{1 - k\,r^2} \qquad R_{1221} = -\frac{r^2\,a^2\,(\dot a^2 + k)}{1 - k\,r^2}$$

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Ricci tensor

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$$R_{\mu\nu} = \begin{pmatrix} -\frac{3\ddot{a}}{a} & 0 & 0 & 0 \\ 0 & u\,g_{11} & 0 & 0 \\ 0 & 0 & u\,g_{22} & 0 \\ 0 & 0 & 0 & u\,g_{33} \end{pmatrix}, \qquad u = \frac{a\,\ddot{a} + 2\,(\dot{a}^2+k)}{a^2}$$

Scalar curvature

$$R = \frac{6\,(a\,\ddot{a} + \dot{a}^2 + k)}{a^2}$$

Einstein tensor

$$G_{\mu\nu} = \begin{pmatrix} \frac{3\,(\dot{a}^2+k)}{a^2} & 0 & 0 & 0 \\ 0 & -v\,g_{11} & 0 & 0 \\ 0 & 0 & -v\,g_{22} & 0 \\ 0 & 0 & 0 & -v\,g_{33} \end{pmatrix}, \qquad v = \frac{2\,a\,\ddot{a} + \dot{a}^2 + k}{a^2}$$

FRW metric EOM EOM-2

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FRW metric EOM EOM-2

Non-trivial Christoffel symbols of Schwarzshield-de Sitter type metric

$$\Gamma^0_{01} = \frac{1}{2}\frac{A'}{A}, \qquad \Gamma^1_{00} = \frac{1}{2}\frac{A'}{B}, \qquad \Gamma^1_{11} = \frac{1}{2}\frac{B'}{B},$$

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SdS metric-GC

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▸ SdS metric-GC

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$$\Gamma^0_{01} = \frac{1}{2}\frac{A'}{A}, \quad \Gamma^1_{00} = \frac{1}{2}AA', \quad \Gamma^1_{11} = -\frac{1}{2}\frac{A'}{A}, \quad \Gamma^1_{22} = -rA, \quad \Gamma^1_{33} = -rA\sin^2\theta,$$

$$\Gamma^2_{12} = \frac{1}{r}, \quad \Gamma^2_{33} = -\sin\theta\cos\theta, \quad \Gamma^3_{13} = \frac{1}{r}, \quad \Gamma^3_{23} = \cot\theta.$$

Non-trivial components of curvature tensor are:

$$R_{0101} = \frac{1}{2}A'', \quad R_{0202} = \frac{r}{2}AA', \quad R_{0303} = \frac{r}{2}AA'\sin^2\theta,$$

$$R_{1212} = -\frac{r}{2}\frac{A'}{A}, \quad R_{1313} = -\frac{r}{2}\frac{A'}{A}\sin^2\theta, \quad R_{2323} = r^2(1-A)\sin^2\theta.$$

The Ricci tensor is diagonal and its components are:

$$R_{00} = \frac{1}{2}AA'' + \frac{1}{r}AA', \qquad R_{11} = -\frac{1}{2}\frac{A''}{A} - \frac{1}{r}\frac{A'}{A},$$

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The scalar curvature is

$$R = -A'' - \frac{4}{r}A' - \frac{2}{r^2}A + \frac{2}{r^2}.$$

The Einstein tensor is presented as follows:

$$G_{00} = -\frac{A(r)A'(r)}{r} - \frac{A(r)^2}{r^2} + \frac{A(r)}{r^2}, \qquad G_{11} = \frac{A'(r)}{rA(r)} - \frac{1}{r^2 A(r)} + \frac{1}{r^2},$$

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SdS metric-C:B=1/A

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