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GRADIENT OF MEAN MASS OF A STAR IN THE FIELD OF GLOBULAR CLUSTERS

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SUMMARY: Based on the relevant available data, the distributions of masses and apparent magnitudes of stars belonging to globular star clusters are simulated. The simulations are aimed at examining the influence of the mass-segregation phenomenon on the surface-density profile. It is found that a minor correction should be introduced in order to infer the profile of the surface density from that of the surface brightness.

Key words. globular clusters: general – Stars: fundamental parameters (masses)

1. INTRODUCTION

Globular star clusters (in further text GCs) are among the most interesting objects in the Universe. It is well known that, in our own Galaxy only, there are about 150 such clusters. They are generally very old (more than 1×10^{10} years) and the global relaxation time is approximately equal to the age. However, due to the steep density gradient, the local relaxation times can differ significantly from the global one, so that those typical for the central parts are significantly shorter than the global time. As a consequence, more massive stars should be predominantly concentrated towards the centre, whereas the massive stars which should be predominantly found towards the periphery. This phenomenon is known as mass segregation. However, since the masses of stars are connected with their luminosities (magnitudes), one can expect the profile of the surface brightness of a GC to be affected by the mass segregation. There is a formula which describes this effect (Ninković and Valjarević 2007). In the present paper its application is extended by simulating GCs. The aim is to establish the general trend in the correction of the surface brightness in order to obtain the real dependence of the surface density on the distance to the cluster centre in projection. Throughout the present paper the author follows the usual assumptions that a typical GC is spherically symmetric and in a steady state, as well as that the light emitted by a GC is contributed by its stars only.

2. GENERAL CONSIDERATION OF MASS-SEGREGATION INFLUENCE ON SURFACE-BRIGHTNESS PROFILE

The formula from the paper by Ninković and Valjarević (2007), applied and examined here, is

$$\sigma = \frac{\bar{\mathfrak{m}}}{\bar{m}_{\rm nor}} I \tag{1}$$

where σ is the surface density, $\bar{\mathfrak{m}}$ is the mean mass of a star, $\bar{m}_{\rm nor}$ is the mean normed apparent magnitude of a star and I is the surface brightness. All the four quantities are functions of $\tilde{r}$ - the distance to the cluster centre in the projection to the tangential plane. Since the apparent magnitude is not an additive quantity, its mean value at a given distance in projection is divided by the total number of stars within a GC. For this reason it is said to be normalized.

It is clearly seen from Eq. (1) that if the mean mass and the mean normalized apparent magnitude were constant, i. e. independent of $\tilde{r}$, then the surface density would be proportional to the surface brightness; this could be also the case if only the ratio $\bar{\mathcal{M}}/\bar{m}_{\rm nor}$ were constant, but such a possibility is improbable. The reason why the two mean values should not be constant lies in the mass segregation. As a consequence of the relaxation process, on the average, more massive stars move more slowly with respect to the centre of the stellar system than the less massive ones. Hence more massive stars tend to concentrate predominantly towards the centre. It is evident that the duration of the relaxation time plays a decisive role. In the systems where the relaxation time exceeds the age significantly, like galaxies, it is quite reasonable to assume the surface density to be proportional to the surface brightness, but in GCs, at least in those belonging to the Milky Way, the global relaxation times are comparable to their ages and, due to the rather high concentration of their matter towards their centres, the relaxation time, for instance in the core, can be significantly shorter than the global one. This is the reason why the mass segregation should be expected to take place within a GC and, consequently, why the observed surface brightness of a GC should be corrected in order to derive the profile of the surface density. Eq. (1) offers such a possibility. Its application requires to examine if there is any correlation between the apparent magnitude of cluster stars and their positions in the tangential plane. If the mass segregation does take place, such a correlation would necessarily occur, as apparent magnitudes and masses of the stars are related to each other. Therefore, the dependence of the ratio multiplying the surface brightness in (1) on $\tilde{r}$ will be examined here. Since the data in real GCs are not available, the only possibility is to carry out corresponding simulations.

The mean apparent magnitude at a given distance will be found from the distributions of stars in apparent magnitude and position (more precisely, the distribution of their projections onto the tangential plane).

Determination of the numerator in the fraction - the mean mass of a star - is more problematic. The reason is clear; unlike the apparent magnitude which is observable, the mass of a star is determined indirectly. The easiest way to estimate the mass of a star is to use its luminosity. The mass-luminosity relation assumed in the present paper was that proposed by Angelov (1993). However, its straightforward application to a GC is barely acceptable because i) it is valid for stars of the Main Sequence (MS), whereas a typical GC contains many evolved stars, and only late-type dwarfs are found along the MS (Fig. 1), and ii) its origin is empirical and based on the data on stars from the Solar neighbourhood, i.e. stars with chemical composition significantly different from that of a GC. Therefore, it is necessary to adapt the assumed mass-luminosity relation to the chemical composition of a GC. This adaptation mainly concerns the abundance of metals (elements heavier than hydrogen and helium) because only their fraction in the total mass can differ significantly from star to star. It is done following Čuljić (2001). Of course, throughout the present paper the usual assumption, that all stars of a GC have the same chemical composition and the same age, is adopted.

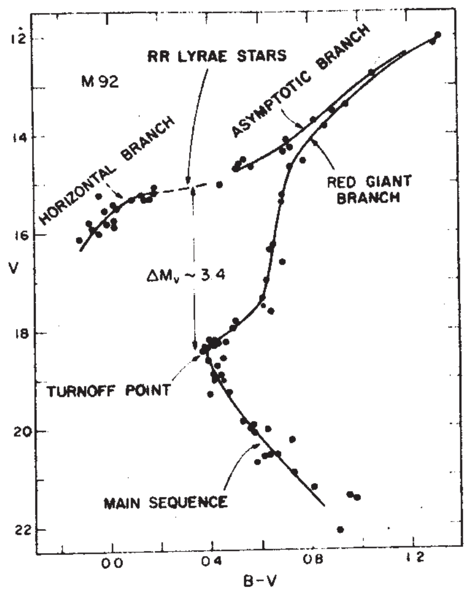

Fig. 1. *Schematic presentation of HR diagram for a typical globular cluster (taken from Lightman and Shapiro 1978).*

As seen from Fig. 1, the stars beyond the MS belong to different branches: red giants, asymptotic branch, horizontal branch and variable stars. Since they spend a shorter time on MS than those stars which are still there, their initial mass must be larger. In fact, they have already begun to lose their mass (e. g. Bressan et al. 1993). In addition, there are stellar remnants: black holes, neutron stars and white dwarfs. Very massive black holes are rather rare so that, except for some special cases, they cannot change the general view. The contribution of all these objects to the luminosity of a GC is usually thought to be negligible (e. g. Binney and Merrifield 1998 - p. 361). However, as mentioned above, since the apparent magnitude is not additive, their contribution to the normalized apparent magnitude should not be neglected.

The luminosity distribution of stars in GCs, taking into account the dependence on the metal abundance, is studied following the data given by Binney and Merrifield (1998 - p. 357) and McClure (1986). In view of the mass-luminosity relation assumed above, this luminosity function can be roughly reduced to Salpeter's (1955) mass function. These mass values are initial, that is MS, masses of stars.

However, in an old stellar system, like a GC, there are many stars beyond MS for which the mass changes must be taken into account. The mass losses before reaching the final phase (stellar remnant) are, as a rule, not high (e.g. Binney and Merrifield 1998 - p. 361) and, therefore, they can be easily estimated. As for the remnants, their masses are estimated on the basis of the corresponding initial masses following the dependence given by Weidemann (1990). In this way the mass-luminosity relation is obtained in the present paper. Besides, there is the dissipation effect which particularly affects low-mass stars. The percentage of escaping stars is estimated by using a formula given by Kholopov (1981 - Eq. (9.29)). This formula indicates that for a GC with global relaxation time approximately equal to its age the actual total number of stars is almost equal to the initial one.

3. SIMULATIONS OF GLOBULAR CLUSTERS

In order to simulate a GC one needs the following data which describe a GC as a whole: integrated apparent magnitude, distance modulus, core radius, tidal (or limiting) radius (both radii in arcminutes), age, chemical composition and total number of stars. It is also taken that a set of cluster stars with measured apparent magnitudes and positions with respect to the cluster centre is available. Since for a GC the spherical symmetry is assumed, the position means distance of a star to the cluster centre in projection in arcminutes. By dividing the actual apparent magnitude of a star by the total number of stars one obtains its normalized apparent magnitude. Known the distance modulus, the relation between the absolute magnitude and mass becomes the relation connecting the mass and apparent magnitude so that to every star with known apparent magnitude one can assign the corresponding mass value.

For a majority of GCs belonging to the Milky Way, the mass for which a star can be still along MS is about one solar mass; in view of the different chemical composition, the corresponding luminosity should be slightly less than that of the Sun. By applying the distance modulus, the given absolute magnitude is converted into the apparent one. The bolometric correction is also taken into account following Binney and Merrifield (1998 - p. 110).

In the simulation, a GC is represented by a sample of stars. Of course, the sample must be unbiased. To achieve this, one applies Salpeter's mass distribution. Clearly, for MS stars these are their actual masses, otherwise, the case has been already described. What follows from Salpeter's law is the fraction of stars within a given interval of initial masses.

Simulation I: $m_{\rm int} = 10.8$, distance modulus equal to 15.5, $r_c = 0.5'$, $r_t = 17'$, age equal to 12 billion years, hydrogen mass fraction equal to 0.75, metal fraction 0.0001 and total number of stars 100 000.

The apparent magnitudes of the sample stars are plotted versus their distance to the centre in projection (Fig. 2). By applying the Origin software, following the Lorentz curve one can find the dependence of the mean apparent magnitude on the distance (solid curve). The masses of these stars are also plotted versus the distance (Fig. 3). The trend (solid line) is found in the same way. Using these plots it is possible to construct the third one presenting the dependence of the mean mass to mean apparent magnitude ratio on the distance in projection (Fig. 4). In order to insert the values in Eq. (1), the mean apparent magnitudes must be normalized (total number of stars given above). It is clearly seen that the ratio is a decreasing function of the distance.

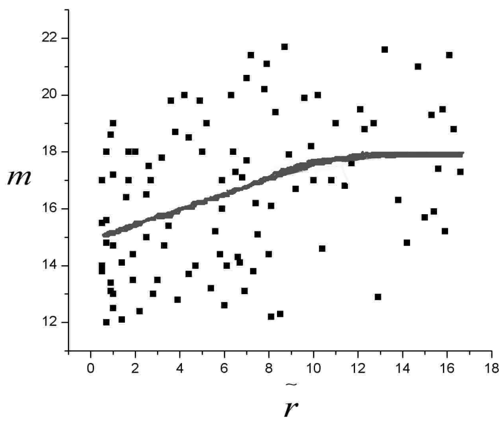

Fig. 2. *Simulation I: apparent magnitudes of cluster stars versus their distance to the centre in projection $\tilde{r}$; $\tilde{r}$ expressed in arcminutes, the solid line gives the dependence of the mean apparent magnitude.*

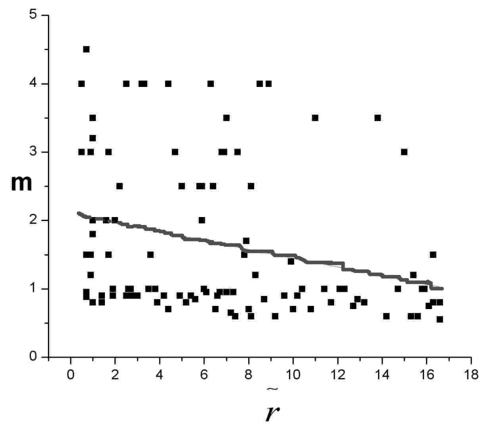

Fig. 3. *Simulation I: masses of cluster stars versus their distance to the centre in projection $\tilde{r}$; $\tilde{r}$ expressed in arcminutes, the mass in solar masses, the solid line gives the dependence of the mean mass.*

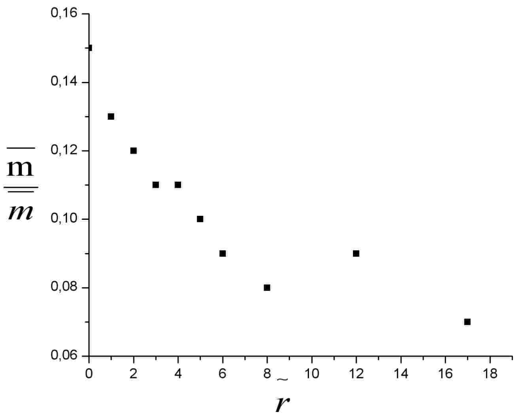

Fig. 4. *Simulation I: the ratio of the mean mass to the mean apparent magnitude versus distance to the centre in projection $\tilde{r}$; $\tilde{r}$ expressed in arcminutes, mass in solar masses.*

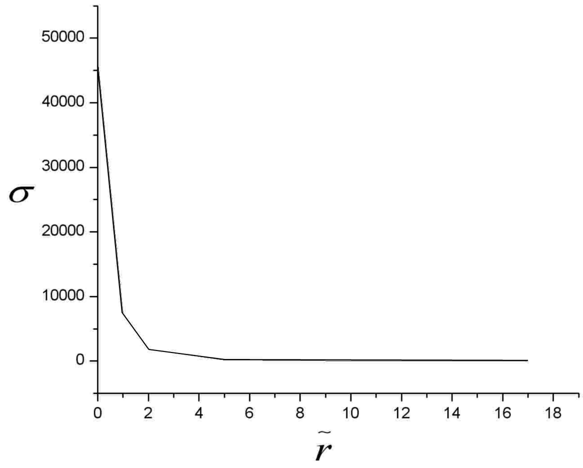

Fig. 6. *Simulation I: surface density of GC versus distance to the centre in projection $\tilde{r}$; $\tilde{r}$ expressed in arcminutes, surface density in solar masses per square arcminute.*

According to Eq. (1), the surface brightness should be multiplied by the ratio given in Fig. 4 with normalized apparent magnitudes. It is assumed that, in the simulated cluster, the surface brightness is given by King's (1962) formula, which requires three parameters to be specified: $m_{\rm int}$, r_c and r_t. The result is presented in Fig. 5. Finally, by using Eq. (1) one obtains the surface density and the corresponding plot is presented in Fig. 6. Due to the fact that the ratio mean mass to mean normalized apparent magnitude is a decreasing function, the decreasing slope for the surface density must be steeper than that for the surface brightness.

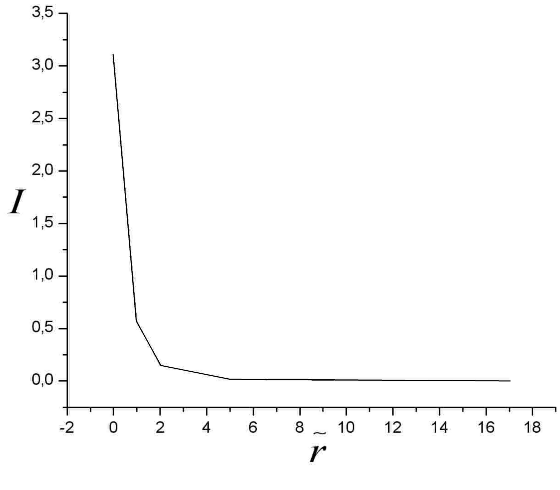

Fig. 5. *Simulation I: surface brightness of GC versus distance to the centre in projection $\tilde{r}$; $\tilde{r}$ expressed in arcminutes, surface brightness in magnitudes per square arcminute.*

The general data are varied and two more simulations are carried out. The corresponding figures are not presented here; the simulations will be briefly mentioned only by specifying their basic data.

Simulation II: $m_{\rm int} = 9.0$, distance modulus equal to 16.0, $r_c = 0.5'$, $r_t = 35'$, age equal to 11.8 billion years, hydrogen mass fraction equal to 0.75, metal fraction 0.0005 and total number of stars 200 000.

Simulation III: $m_{\rm int} = 10.0$, distance modulus equal to 17.0, $r_c = 1.0'$, $r_t = 40'$, age equal to 10.5 billion years, hydrogen mass fraction equal to 0.75, metal fraction 0.001 and total number of stars 300 000.

In all the three cases it is found that the ratio of the mean mass to the mean normalized apparent magnitude of a star is a decreasing function of the distance to the centre in projection.

4. DISCUSSION AND CONCLUSIONS

The results obtained in the present paper clearly demonstrate that the negative gradient of the mean mass of a star in the field of a GC leads to the analogous, but positive, gradient of the mean apparent magnitude. The relation between these two quantities is rather well defined within a GC: for a vast majority of stars the higher the mass, the lower is the apparent magnitude. The exception concerns the stellar remnants, among which the white dwarfs prevail, because, in spite of their still significant masses, such stars have very low luminosities. Simulations in a problem like the one solved here are important since in the field of a typical GC white dwarfs cannot be observed due to their extremely low brightness. However, according to Salpeter's law, assumed here, the white dwarfs are not expected to be very numerous, the red dwarfs, also unobservable due to their very low brightness, are much more numerous. In the present simulations this effect is specially studied and this is the reason why the simulated sample (Fig. 2) shows a rather strong increase in the mean apparent magnitude. On the other hand, the mass, unlike the apparent magnitude, is not observable and for this reason simulations like the present ones can be important for the purpose of introducing a model of a typical GC in which the dependence of the mean mass of a star on the distance to the centre (not in projection) could be proposed to be tested afterwards in the analysis of the observational results.

As for the mass-luminosity relation, it is applied here following Angelov (1993) and Čuljić (2001), where the fraction of metals is varied since this is the crucial parameter. The most important to be said is that the chemical composition affects the mass estimation via luminosity only quantitatively, not qualitatively, but the influence is not too strong, especially if borne in mind that, among the GCs of the Milky Way, those with a high metallicity are very rare. The present simulations have the objective to reflect the situation in more or less typical GCs, i.e. objects of low metallicity.

Thus, as a final conclusion it may be said that the effect of mass segregation should be observed in the field of a typical GC through the correlation between the apparent magnitude distance of a star to the cluster centre in projection. However, to have a complete information on the behaviour of the surface density one must examine the gradient, in projection of the mean mass of a star. It seems that this effect is not too strong and, as a first approximation, one may neglect it. Of course, the modern observational facilities, including our future station at Vidojevica as well, are expected to help in the verification of this finding and offer a reliable observational basis for more sophisticated models of GCs where the effect of mass segregation would be more thorough by taken into account.

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REFERENCES

Angelov, T.: 1993, *Bull. Obs. Astron. Belgrade*, **148**, 1.

Binney, J. and Merrifield, M.: 1998, Galactic Astronomy, Princeton University Press, Princeton, New Jersey.

Bressan, A., Fagotto, F., Bertelli, G. and Chiosi, C.: 1993, *Astron. Astrophys. Suppl. Series*, **100**, 647.

Čuljić, A.: 2001, Magistarski rad, Univerzitet u Beogradu.

King, I.: 1962, *Astron. J.*, **67**, 471.

Kholopov, P.N.: 1981, Zvezdnye skopleniya, Nauka, glav. red. fiz.-mat. lit., Moskva.

Lightman, A.P. and Shapiro, S.L.: 1978, *Rev. Mod. Phys.*, **50**, 437.

McClure, R.D.: 1986, *Astrophys. J*, **307**, L49.

Ninković, S. and Valjarević, A.: 2007, *Serb. Astron. J.*, **174**, ...

Salpeter, E.E.: 1955, *Astrophys. J*, **121**, 161.

Weidemann, V.: 1990, *Annu. Rev. Astron. Astrophys.*, **28**, 103.

ГРАДИЈЕНТ ПРОСЕЧНЕ МАСЕ ЗВЕЗДЕ У ПОЉУ ЗБИЈЕНОГ ЗВЕЗДАНОГ ЈАТА

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На основу расположивих релевантних података извршена је симулација расподела по масама и привидним величинама за звезде збијених звезданих јата. Сврха ове симулације је да се испита утицај појаве раздвајања маса на зависност површинске густине. Нађено је да треба применити мању поправку да би се из зависности површинског сјаја извела расподела површинске густине.