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DYNAMICS OF TACHYON FIELDS AND INFLATION - COMPARISON OF ANALYTICAL AND NUMERICAL RESULTS WITH OBSERVATION

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SUMMARY: The role tachyon fields may play in evolution of early universe is discussed in this paper. We consider the evolution of a flat and homogeneous universe governed by a tachyon scalar field with the DBI-type action and calculate the slow-roll parameters of inflation, scalar spectral index (n), and tensor-scalar ratio (r) for the given potentials. We pay special attention to the inverse power potential, first of all to $V(x) \sim x^{-4}$, and compare the available results obtained by analytical and numerical methods with those obtained by observation. It is shown that the computed values of the observational parameters and the observed ones are in a good agreement for the high values of the constant X_0. The possibility that influence of the radion field can extend a range of the acceptable values of the constant X_0 to the string theory motivated sector of its values is briefly considered.

Key words. cosmology: theory – cosmological parameters – early Universe

1. INTRODUCTION

The period of the very early universe, despite its short duration, is probably the most challenging one in the universe evolution to be understood. In recent years there has been a lot of evidence from WMAP and Planck observations of the Cosmic Microwave Background (Bennett et al. 2003, Komatsu et al. 2011, Spergel et al. 2003, Planck Collaboration 2014, 2015) that the early universe underwent a period, of inflation, when its expansion rate was accelerating (Guth 1981). Probably the best way to describe the evolution of the early universe is through quantum cosmology (Wiltshire 1996). The standard method to study inflation is to suppose that the inflationary phase is driven by potential energy of the canonical scalar field (inflaton) whose dynamics is described by the standard Lagrangians and the corresponding Klein-Gordon equation (Liddle and Lyth 2000).

In addition to models based on the standard Lagrangian, there is a popular class of models motivated by string theory and non-standard scalar field actions. Among several string motivated models, Dirac-Born-Infeld (DBI) and tachyon models have attracted a great deal of attention (Gibbons 2003, Steer and Vernizzi 2004 and references therein).

In this paper we consider the DBI-type scalar field in a cosmological context as an effective field theory which describes rolling tachyons (Gibbons 2003, Sen 2002a, Sen 2002b). We solve numerically the Friedman equations and calculate slow-roll and observational parameters for a number of the most interesting potentials. The results we obtain are compared with analytical results, where they exist, computed in the slow-roll approximation (Steer and

Vernizzi 2004) and observational data from Planck mission (Planck Collaboration 2014, 2015).

The paper is arranged in the following way. In Section 2 we review the basic ideas of tachyonic inflation. In the next Section 3 we apply the slow-roll conditions to tachyon models of inflation and rescale the Friedman equations and energy-momentum conservation equation to the dimensionless form. In Section 4 we discuss dynamics of inflation and compare numerical results with analytical solutions for two of three considered potentials and observational data. In Section 5 we give a conclusion, and argument and direction for further research.

2. TACHYONIC INFLATION

Traditionally, the word tachyon was used to describe a hypothetical particle which propagates faster than light. In modern physics this meaning of tachyon has been changed. Since the beginning of the string theory, states of quantum fields with imaginary mass (i.e. negative mass squared) were called tachyons. It was believed that such fields permitted propagation faster than light, but soon it was realized that the imaginary mass creates an instability and tachyons spontaneously decay through the process known as tachyon condensation (Sen 2003).

There is no classical interpretation of the "imaginary mass"; however, the instability in the system can be interpreted in the following way. The potential of the tachyonic field is initially at a local maximum rather than a local minimum (like a ball at the top of a hill). A very small perturbation, due to quantum fluctuations, forces the field to roll down (the hill) towards the local minimum. Once the tachyonic field reaches the minimum of the potential, its quanta are not tachyon any more, but rather an "ordinary" particle with a positive mass.

We consider the tachyonic field T minimally coupled to Einstein's gravity. The action is:

$$S = -\frac{1}{16\pi G}\int \sqrt{-g}Rd^4x + S_T, \tag{1}$$

where R is the Ricci scalar, g is the determinant of the metric tensor with components $g_{\mu\nu}$. The tachyon action S_T is:

$$S_T = \int \sqrt{-g}\mathcal{L}(T,\partial_\mu T)d^4x, \tag{2}$$

where:

$$\mathcal{L} = -V(T)\sqrt{1+g^{\mu\nu}\partial_\mu T\partial_\nu T} \tag{3}$$

is the DBI-type Lagrangian density (Sen 2002a, Sen 2002b). The Hamiltonian density is given by:

$$\mathcal{H} = V(T)\left(1+g^{\mu\nu}\partial_\mu T\partial_\nu T\right)^{-1/2}. \tag{4}$$

As it is common, we restrict our consideration on a homogeneous and isotropic space with the Friedmann-Robertson-Walker metrics:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\vec{x}^2, \tag{5}$$

where $a(t)$ is the scale factor of the universe. The tachyon field in this metric can be split into a homogeneous time dependent contribution $T(t)$ and a small $\mathbf{x}$-dependent perturbation $\delta T(t,\vec{x})$ which describes quantum fluctuations of the field T (Steer and Vernizzi 2004). In the following discussion we will focus only on the homogeneous (time dependent) contribution.

It was shown by Sen (2002a, 2002b) and Gibbons (2002) that in a homogeneous and isotropic space the tachyon field behaves like a fluid of positive energy density:

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}}, \tag{6}$$

and negative pressure:

$$P = -V(T)\sqrt{1-\dot{T}^2}. \tag{7}$$

Note that pressure and energy density are $P = \mathcal{L}$ and $\rho = \mathcal{H}$, like in the case of a scalar field with a standard-type Lagrangian.

The Friedman equation takes the standard form:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2}\frac{V}{(1-\dot{T}^2)^{1/2}}, \tag{8}$$

where H is the Hubble parameter and $M_{Pl} = (8\pi G)^{-1/2}$ is the reduced Planck mass. We assume that the space is spatially flat and the cosmological constant is equal to zero.

In addition, the energy-momentum conservation equation:

$$\dot{\rho} = -3H(P+\rho), \tag{9}$$

using Eqs. (6) and (7) is transformed into a second order differential equation:

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0, \tag{10}$$

where the dot denotes the differentiation with respect to time and $V' = dV/dT$.

It is argued that tachyonic inflation cannot be responsible for a sufficiently long period of inflation. It might be possible that tachyonic inflation is responsible for an earlier stage of inflation (Kofman and Linde 2002), at and "around" the Planck scale, which may be important for the resolution of homogeneity, flatness and isotropy problems (Kofman and Linde 2002). At that point, tachyonic inflation, and in particular tachyon field dynamics, may need a quantum and non-Archimedean approach (Dimitrijevic et al. 2016, Djordjevic et al. 2016).

Despite problems, tachyon-driven scenarios remain highly interesting for study. There have been a number of attempts to understand the evolution of the early universe via (classical or quantum) nonlocal cosmological models on both Archimedean and non-Archemedean spaces (Barnaby et al. 2007, Dimitrijevic et al. 2008, Joukovskaya 2007, Moeller and Zwiebach 2002) with interesting results.

3. SLOW-ROLLING TACHYON

As it has been mentioned, inflation is a period in evolution of the early universe when the universe underwent a phase of very fast (exponential) expansion. The solution of the system of Eqs. (8) and (10) does not always describe the accelerated universe. It is possible to impose specific conditions before solving the Friedman equation to ensure that the scale factor of the universe will be drastically increased. The kind of conditions for accelerated expansion are the so-called slow-roll conditions. The slow-roll conditions can be easily set by introducing slow-roll parameters.

The slow-roll parameters can be defined in different ways. In this paper we will use the definition given by Schwarz et al. (2001) so that the slow-roll parameters are derivatives of the Hubble parameter (H) with respect to the number of e-foldings (N):

$$\epsilon_{i+1} \equiv \frac{d\ln|\epsilon_i|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}, \tag{11}$$

where H_* is the Hubble parameter at some chosen time. The number of e-folds is:

$$N(t) = \int_{t_i}^{t_e} H(t)dt, \tag{12}$$

where t_i is the time when counting of e-folds began and t_e is the time at the end of inflation.

The first two parameters are:

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{1}{H}\frac{\ddot{H}}{\dot{H}} + 2\epsilon_1. \tag{13}$$

The conditions for slow-roll inflation are satisfied when $\epsilon_1 < 1$ and $\epsilon_2 < 1$, and inflation ends when any of them exceeds unity.

Using the Friedmann acceleration equation:

$$\dot{H} = -\frac{1}{2M_{Pl}^2}(P+\rho), \tag{14}$$

and Eq. (8), the slow-roll parameters can be expressed as:

$$\epsilon_1 = \frac{3}{2}\dot{T}^2, \quad \epsilon_2 = 2\frac{\ddot{T}}{H\dot{T}}. \tag{15}$$

Here it is convenient to introduce a constant dimensionless ratio X_0 which characterizes flatness of the potential $V(T)$ close to its peak (Fairbairn and Tytgat 2002):

$$X_0 = \frac{\lambda T_0^2}{M_{Pl}^2}, \tag{16}$$

where λ is a constant and potential $V(T)$ satisfies the following properties:

$$V(0) = \lambda, \quad V'(T>0) < 0, \quad V(|T| \to \infty) \to 0. \tag{17}$$

In the string theory motivated potentials the constant λ is defined as (Gerasimov and Shatashvili 2000)

$$\lambda = \frac{M_s^4}{g_s(2\pi)^3}, \tag{18}$$

where M_s is the string mass and g_s is the string coupling.

It is convenient to rescale the field T and the Friedmann equations (8), (10) and (14) by introducing a constant T_0. Like the constant λ, the constant T_0 has its origin in string theory (Fairbairn and Tytgat 2002). Additionally, the cosmic time is rescaled as $\tau = tT_0$ and we introduce the dimensionless quantities:

$$x = \frac{T}{T_0}, \quad U(x) = \frac{V(x)}{\lambda}, \quad \tilde{H} = \frac{H}{T_0}. \tag{19}$$

The dimensionless Friedman equation (8) can be written now as:

$$\tilde{H}^2 = \frac{X_0^2}{3}\frac{U(x)}{\sqrt{1-\dot{x}^2}}. \tag{20}$$

The energy-momentum conservation equation (10) takes the form:

$$\ddot{x} + X_0\sqrt{3U(x)(1-\dot{x}^2)^{3/2}}\dot{x} + \frac{(1-\dot{x}^2)}{U(x)}\frac{dU(x)}{dx} = 0, \tag{21}$$

and the Friedman acceleration equation (14) is:

$$\dot{\tilde{H}} = -\frac{X_0^2}{2}(\tilde{P}+\tilde{\rho}), \tag{22}$$

where the dot denotes a derivative with respect to τ and:

$$\tilde{\rho} = \frac{U(T)}{\sqrt{1-\dot{x}^2}}, \quad \tilde{P} = -U(x)\sqrt{1-\dot{x}^2}. \tag{23}$$

The system of equations written in a dimensionless form is much easier to solve numerically. The dimensionally reduced form of equations helps in standardizing equations and makes them independent of variable scales. Besides, it is much easier to recognize which mathematical and numerical techniques should be applied, the number of free parameters is reduced and the equations can be solved drastically faster.

3.1. Conditions for tachyonic inflation

The main condition that is required for inflation is an accelerated expansion (Liddle and Lyth 2000):

$$\frac{\ddot{a}}{a} \equiv \tilde{H}^2 + \dot{\tilde{H}} = \frac{X_0^2}{3}\frac{U(x)}{\sqrt{1-\dot{x}^2}}\left(1-\frac{3}{2}\dot{x}^2\right) > 0. \tag{24}$$

The condition in Eq. (24) requires that $\dot{x}^2 < \frac{2}{3}$. Following the standard procedure for standard single field inflation (Liddle and Lyth 2000, Steer and Vernizzi 2004) the slow-roll conditions are:

$$\ddot{x} \ll 3\tilde{H}\dot{x}, \quad \dot{x}^2 \ll 1. \tag{25}$$

Bearing in mind Eqs. (19) and (6 - 8), the condition (24) and relations (25) simplify the problem with indefinity or imaginarity which appears in Eqs. (6) - (8) when $\dot{T}^2 \geq 1$. The case when $\dot{T}^2 \geq 1$ could be important when considering a pure (tachyon) field dynamics (Rizos and Tetradis 2012, Dimitrijevic et al. 2016, Djordjevic et al. 2016).

During the period of inflation the Friedmann equation (20) and Eq. (21) take the approximate forms:

$$\tilde{H}^2 \sim \frac{X_0^2}{3} U(x), \quad \dot{x} \sim -\frac{1}{3\tilde{H}} \frac{U'(x)}{U(x)}. \tag{26}$$

Combining Eqs. (15) and (26) the slow-roll parameters can be written in terms of the tachyon potential and its derivatives with respect to x:

$$\epsilon_1 \simeq \frac{1}{2X_0^2} \frac{U'^2}{U^3}, \quad \epsilon_2 \simeq \frac{1}{X_0^2} \left(-2\frac{U''}{U^2} + 3\frac{U'^2}{U^3} \right). \tag{27}$$

In terms of tachyon field and potential, the number of e-folds Eq.(12) is given by:

$$N(x) = X_0^2 \int_{x_i}^{x_e} \frac{U(x)^2}{|U'(x)|} dx, \tag{28}$$

where $x_e = x(\tau_e)$ and $x_i = x(\tau_i)$.

3.2. Observational parameters

During the last twenty years, cosmology and inflation have been transformed from the mainly theoretical to the observationaly supported field of science. Many research missions provided a lot of observational data and a possibility to test and compare predictions of cosmological and inflationary models with the real data. The modern cosmology is nowdays more and more connected with modern Astronomy in many aspects.

Unfortunately, the slow-roll parameters of inflation cannot be directly measured. However, the results of Planck Collaboration (2014) and previous missions provided limits on other parameters that can be both measured and calculated in the models.

The most important observable parameters are the scalar spectral index (n) and the tensor-scalar ratio (r). In terms of slow-roll parameters (13) tensor-scalar ratio can be written as (Peiris et al. 2003):

$$r = 16\epsilon_1, \tag{29}$$

and the scalar spectral index is:

$$n = 1 - 2\epsilon_1 - \epsilon_2, \tag{30}$$

where ϵ_1 and ϵ_2 are the slow-roll parameters at some particular moment at the beginning of inflation (i.e. $t = t_i$ in Eq. (12) and $x = x_i$ in Eq. (28)).

Current constraints on the tensor-scalar ratio and scalar spectral index given by Planck Collaboration XX (2015) are

$$n = 0.9655 \pm 0.0062, \tag{31}$$

$$r \leq 0.11. \tag{32}$$

In the following section computations of the observational parameters n and r are presented. The measured constraints for r and n are used to set limits on the dimensionless constant X_0 and to verify if the given model and tachyon potential correspond to observational data.

4. DYNAMICS OF INFLATION

In this section of the paper we review some new results for two common potentials: $V(x) = \lambda e^{-x}$ and $V(x) = \lambda / \cosh(x)$, and present completely new analytical and numerical results for the inverse power law potential $V(x) = \lambda / x^4$ in more details.

For each potential we calculate the slow-roll parameters, scalar spectral index, and tensor-scalar ratio and compare firstly numerical and analytical results, and compare these results with the observational data. Classical and quantum dynamics of tachyon fields on Archemedean and non-Archimedean spaces have been considered in detail in the recent papers (Dimitrijevic et al. 2016, Djordjevic et al. 2016).

4.1. Analytical method

In the analytical approach the approximative expression (27) is used. The standard procedure is followed: for a given potential, the quantity ϵ_1 is computed as a function of x, then x_e is estimated from the slow-roll condition $\epsilon_1(x_e) = 1$.

Then, for a chosen set of parameters N and X_0 the lower limit x_i of the integral in Eq. (28) is computed. Now, the observable parameters $n(x_i)$ i $r(x_i)$ can be calculated.

The Friedmann equation (26) in the slow-roll regime is solved. In the case of the power law potential $U(x) = 1/x^4$ the analytical solution for a rescaled tachyon field is given by:

$$x(\tau) = C_1 e^{\frac{4\tau}{X_0 \sqrt{3}}}. \tag{33}$$

The solution $x(t)$ is substituted in Eq. (15) and time evolution of slow-roll parameters is calculated. The slow-roll parameters for the potential $U(x) = 1/x^4$ are:

$$\epsilon_1(\tau) = \epsilon_2(\tau) = \frac{8C_1^2}{X_0^2} e^{\frac{8\tau}{X_0 \sqrt{3}}}. \tag{34}$$

4.2. Numerical method

It was shown by Steer and Venizzi (2004) that the slow-roll parameters (27) for the tachyon field are the same as in the standard single field inflation up to the first order. For any known potential those parameters can be calculated from Eq. (27), and there is no need to solve differential equations (20) and (21) to find the solution for the field and Hubble parameter. However, there are quite important models (for example multiple fields, etc.) when this approach cannot be used, and the Eq. (12) has to be used. The system of equations (20) and (21) is not always solvable analytically, so numerical methods have to be used instead. Once developed, numerical algorithms can be modified for different and more complicated models. In this paper the numerical algorithm we developed is tested by applying it to the potentials that have been already analytically solved ($U(x) = \exp(-x)$, $U(x) = 1/\cosh(x)$).

Numerical method is based on solving dimensionless equations without slow-roll approximation. Eqs. (20)-(22) are solved for the initial conditions $x(0) = x_i$, and $\dot{x}(0)$ is limited by the inflationary condition (25) to a very small value ($\dot{x}(0) \to 0$). The solutions $x(\tau)$ and $\tilde{H}(\tau)$ of these equations are used to calculate the slow-roll parameters (13) and (15). A similar standard procedure to calculate ϵ_1 and ϵ_2 is applied as well. Due to the fact that the solution $x(\tau)$ is time dependent we have to use the number of e-foldings given by Eq. (12). Neglecting the errors that appeared due to numerical calculations, the lower limit τ_i of the integral in Eq. (12) approximately corresponds to the initial rescaled tachyon field $x_i = x(\tau_i)$ in Eq. (28), as expected. Due to the difference between numerical and analytical results when $\tau \to 0$ we subtract a small number of the first e-folds (10^{-3} - 10^{-2}) and consider a negligibly shorter period of inflation.

Finally, the tensor-scalar ratio $r(x_i)$ and spectral index $n(x_i)$ are calculated for the corrected number of e-foldings.

4.3. Results and Discussion

The main results we presented here are calculated in C/C++ by using standard numerical algorithms: the finite difference method for differentiation, the Simpson's method for integration, the bisection method for finding the root of functions and the fourth order Runge-Kutta method for solving differential equations (Galassi et al. 2009, Press et al. 2007).

The Friedman equation is solved numerically and the observational parameters are calculated for the potential $V(x) = \lambda/x^4$. We also calculate analytically the observational parameters for this potential in the slow-roll approximation and compare both of them. The results are calculated for a sample of sets (N, X_0) where N can take any integer value $45 \leq N \leq 75$, and for X_0: $5 \leq X_0 \leq 25$.

The results we obtained are shown in Figs. 1 and 2.

It can be seen easily from the plots that the numerical solutions correspond to analytical ones very well. The solutions for the Hubble parameters ($\tilde{H}$), Fig. 2, are almost identical and the solutions for the tachyon field are in a good correlation during inflation (Fig. 1). As the universe evolves toward the end of inflation the difference between analytical and numerical results grows. This difference is due to the fact that the analytical result is obtained with the slow-roll approximation which is valid only during inflation.

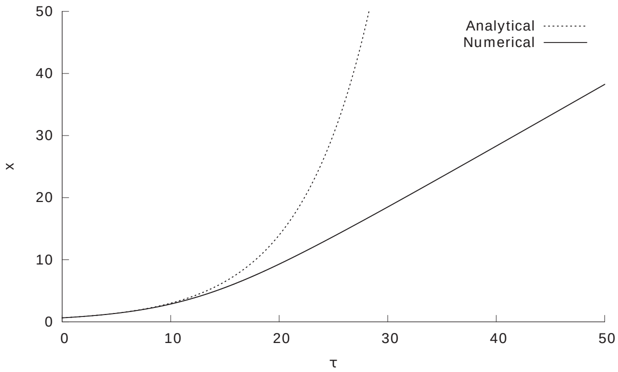

Fig. 1. *Analytical and numerical solution for dynamics of tachyon field x for the potential $U(x) = x^{-4}$ ($N = 65$, $X_0 = 15$).*

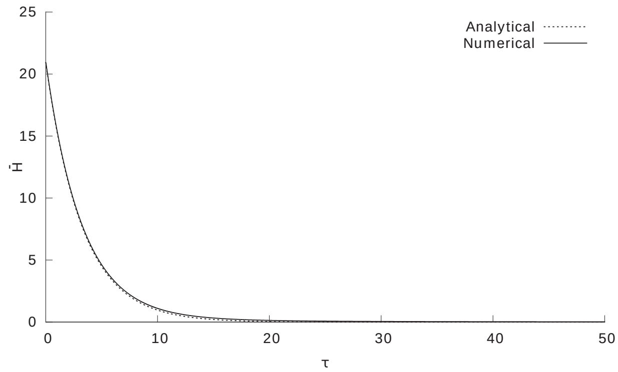

Fig. 2. *Comparison of analytical and numerical results for time evolution of the dimensionless Hubble parameter. The potential and parameters are the same as in Fig. 1. The two solutions are overlapping and are almost indistinguishable in this plot.*

However, it was noticed that there is a significant difference in numerical and analytical solutions for x and $\tilde{H}$ at the very beginning, i.e. when $\tau \ll 1$. This difference originates in the initial condition ($\dot{x}(0) \approx 0$) which exists in numerical solutions and it is neglected in the slow-roll approximation. After this short disturbance at the beginning, the solutions become the same and further evolution does not depend on this initial condition.

The corresponding slow-roll parameters are calculated and the results are shown in Figs. 3-5.

Numerical results for the slow-roll parameter show (Fig. 3) that the inflation lasts a few e-foldings longer than it is in the analytical case with slow-roll approximation. Besides, for the fixed N the inflation lasts longer if the value of X_0 is larger. The results for the three values of X_0 ($X_0 = 5$, $X_0 = 15$ and $X_0 = 25$; $N = 65$) are shown in Fig. 5.

The results computed (both numerically and analytically) for observational parameters n and r are shown in Fig. 6. It is interesting to compare these results with the results obtained for $U(x) = 1/\cosh(x)$ (Fig. 7) and $U(x) = \exp(-x)$ (Fig. 8) which are presented in Bilic et al. (2016a). It can easily be noticed that the reciprocal cosine hyperbolic and exponential potential better agree with the observational data (31 - 32) than the inverse quartic one, i.e. the model with the inverse quartic potential is less favorable.

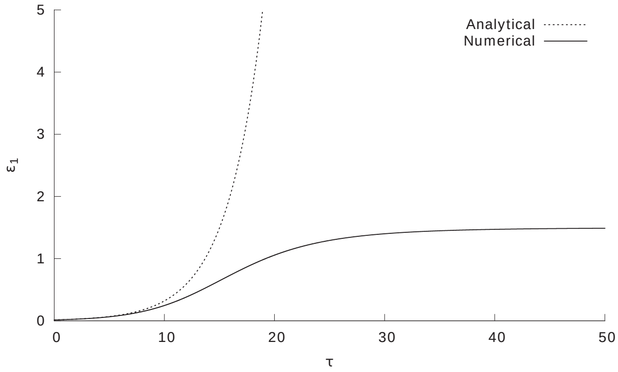

Fig. 3. *The slow-roll parameter $\epsilon_1(\tau)$ as a function of time τ for $N = 65$ and $X_0 = 15$.*

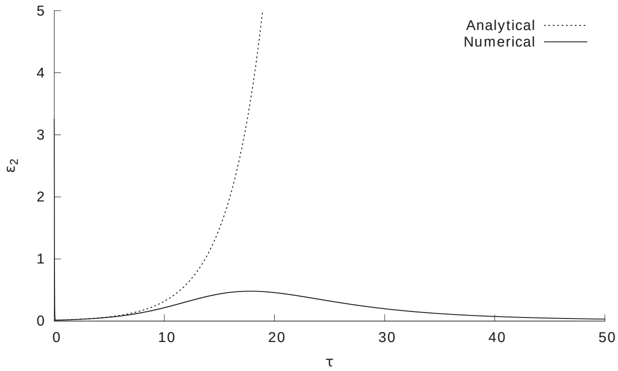

Fig. 4. *The slow-roll parameter $\epsilon_2(\tau)$ as a function of time τ for the same parameters as in Fig. 3.*

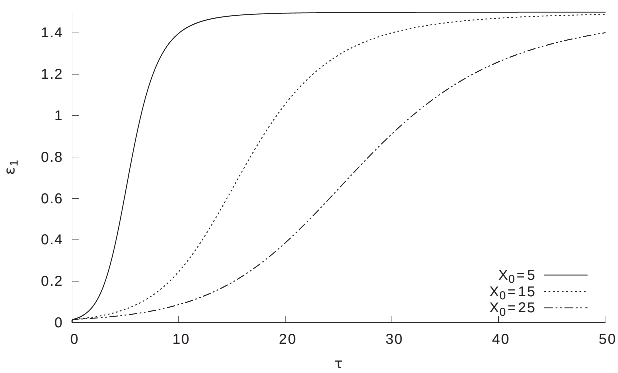

Fig. 5. *The slow-roll parameter $\epsilon_1(\tau)$ as a function of time τ for $N = 65$ and various values of X_0.*

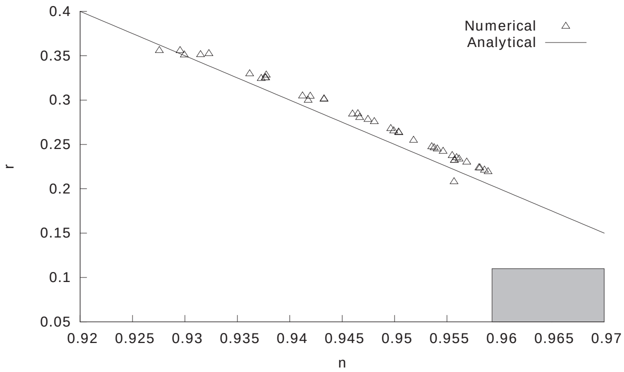

Fig. 6. *Comparison between the values in (n, r) parameter space calculated numerically (triangles), as well as analytically (line), and the corresponding observational constraints obtained by Planck Collaboration 2015 (shaded square region).*

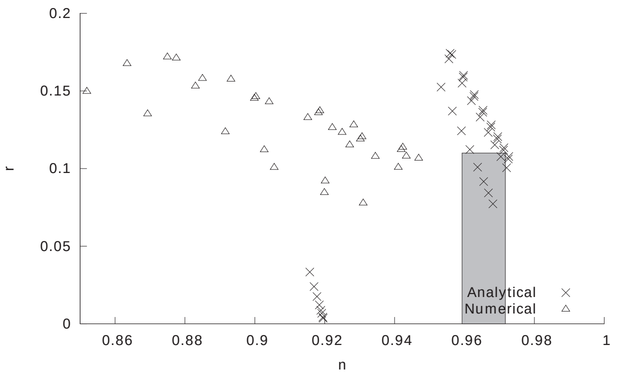

Fig. 7. *The observational parameters n and r for $U(x) = 1/cosh(x)$. Comparison between numerical (triangles) and analytical (crosses) results.*

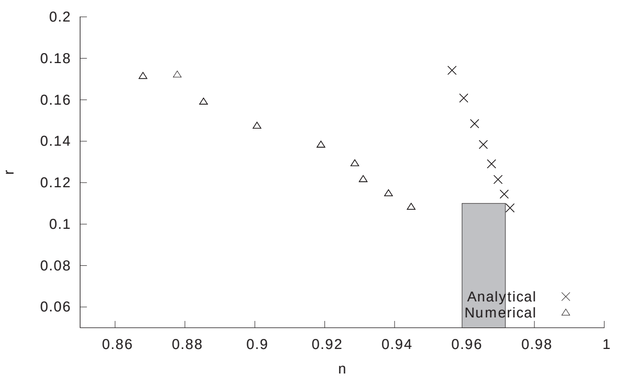

Fig. 8. *Similar as in Fig. 7. The observational parameters n and r for the potential $U(x) = exp(-x)$.*

On the other hand, this inverse quartic model has been studied in another context. It was proposed by Bilic and Tupper (2014) to construct a model based on Randal Sundrum II (RSII) model, introducing a radion field, which parametrizes the interbrane distance. In addition, the fifth coordinate can be treated as an additional dynamical scalar field. In the first approach, it appeared that the scalar field can be transformed to a new field, which corresponds to the tachyonic field with the exactly inverse quartic potential, $V(T) \sim T^{-4}$. The inclusion of effects of both the radion field and tachyon field with the inverse quartic potential could give a better estimation for n and r in accordance with the observational data.

As it has been expected, the results for the potential $U(x) = x^{-4}$ are not in an excellent agreement with the observed values (Eqs. (31) - (32)) of the tensor-scalar ratio (r) and scalar spectral index (n). It is worth to stress that the computed values of the observational parameters and the observed ones are in a very good agreement only when $X_0 \gg 1$; more precisely an excellent agreement is achieved for $X_0 \sim 500$. However, those results are out of the range of string motivated values for X_0 (Steer and Vernizzi 2004) and will be studied elsewhere.

5. CONCLUSION

In this paper we considered the solution of the energy-momentum conservation equation (10) and compute tensor-scalar ratio (r) and scalar spectral index (n) numerically and analytically (with slow-roll approximation). We compared them with observational results, mostly from the Planck Collaboration XXII (2014).

The results we have obtained and presented here confirm that the numerical method we used is adequate and in a good agreement with the analytical ones. However, the values of observational parameters, n and r, for string motivated potentials in "pure" tachyon inflation are not quite in the measured range for those parameters. A good agreement can be achieved in the beginning of inflation.

The numerical method we used can be easily accommodated for any other form of potential, for different models, which makes our approach a pretty general one. It can be used to compute the slow-roll parameters in models based on more complicated potentials or in models where the potential is unknown, even when the equations cannot be solved analytically due to any reason.

At the end, it should be emphasized that we have started to implement this method to the RSII model and studied the inflationary scenario based on the tachyon field with $V(T) \sim T^{-4}$ coupled with the radion. The complexity of the model and of the corresponding system of nonlinear differential equations does not allow an analytical solution. Work on these RSII tachyon-radion "hybrid" models is in progress and will be published elsewhere (Bilic et al. 2016b).

Finally, it seems that the origin of inflation has to be a quantum process and a very challenging one. Recent results (Dimitrijevic et al. 2016, Djordjevic et al. 2016) are a very good base for an extended quantum vs classical approach and it could shed more light on processes, closer to the Planck scale, on Archimedean and non- Archemedean spaces and beyond.

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ДИНАМИКА ТАХИОНСКИХ ПОЉА И ИНФЛАЦИЈЕ - УПОРЕЂИВАЊЕ АНАЛИТИЧКИХ И НУМЕРИЧКИХ РЕЗУЛТАТА СА ПОСМАТРАЊИМА

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Оригинални научни рад

Дискутована је могућа улога тахионских поља у еволуцији раног свемира. Разматрамо еволуцију равног и хомогеног свемира вођену тахионским скаларним пољем са одговарајућим дејством Дирак-Борн-Инфелд типа и рачунамо параметре инфлације (параметри спорог котрљања), скаларни спектрални индекс (n) и тензор-скалар однос (r) за различите потенцијале. Посебна пажња је посвећена потенцијалима степеног закона, пре свега $V(x) \sim x^{-4}$, и упоређивању резултата добијених аналитичким и нумеричким методом са посматрањчким резултатима. Показано је да се израчунате вредности посматрачких параметара слажу са измереним вредностима параметара само за велике вредности константе X_0. Укратко је разматрана могућност да утицај радионског поља прошири опсег вредности константе X_0 и на оне вредности које предвиђа теорије струна.