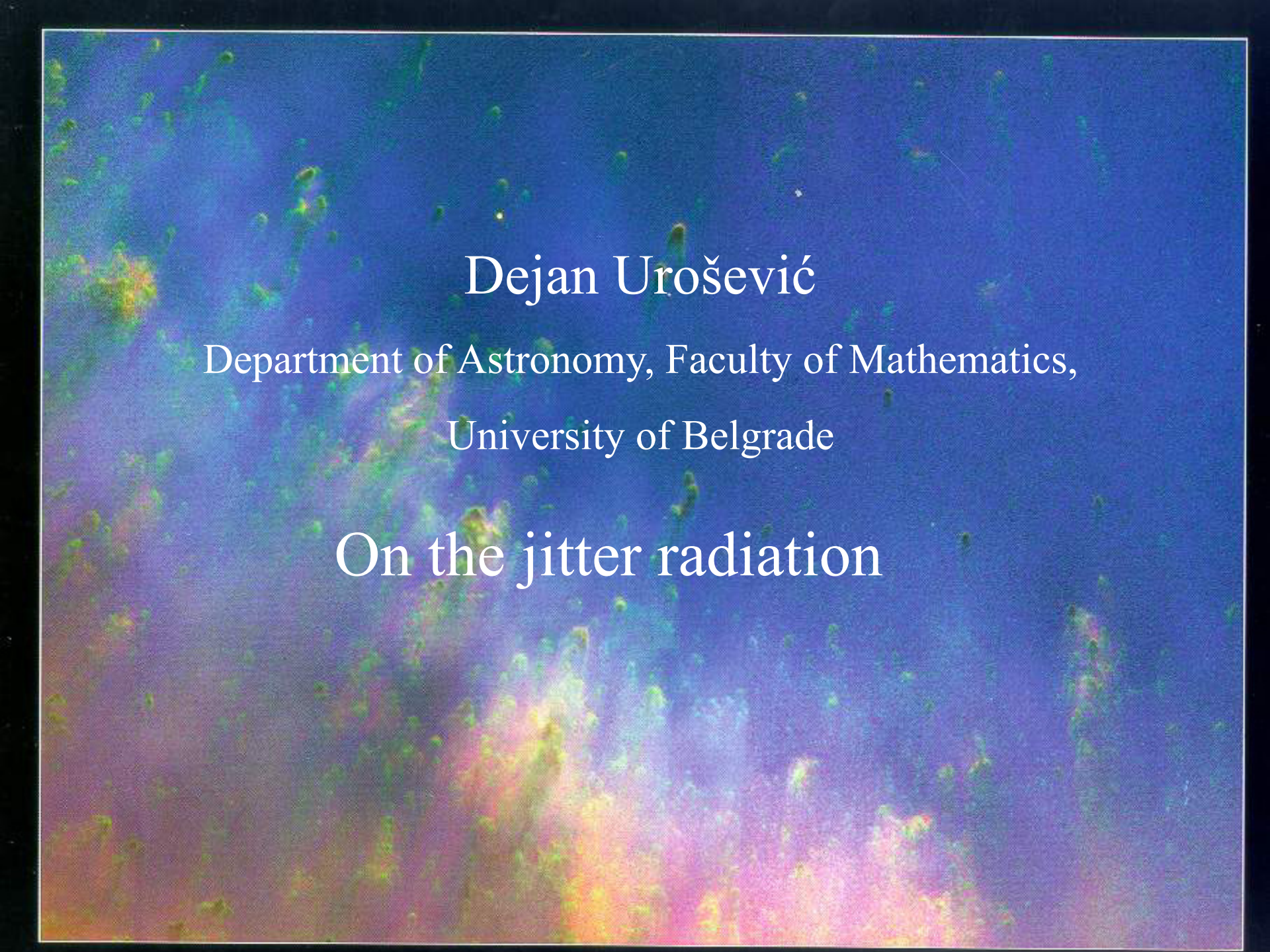




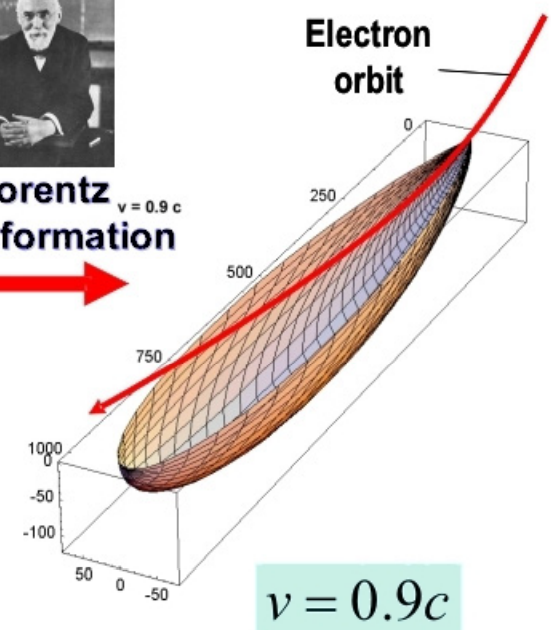
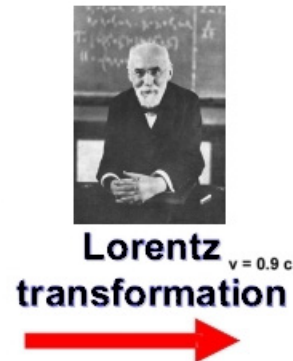
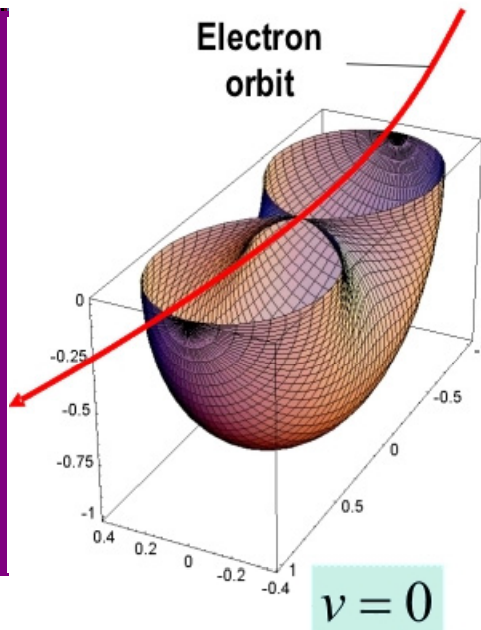
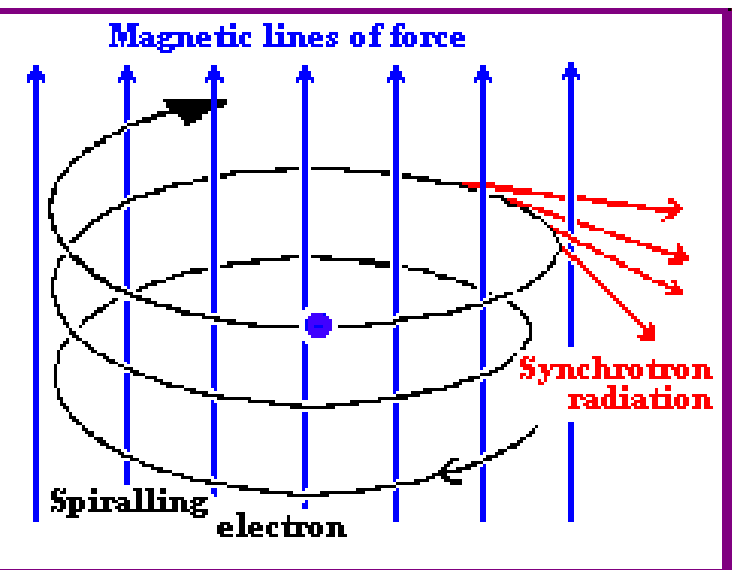
Dejan Urošević

Department of Astronomy, Faculty of Mathematics,
University of Belgrade

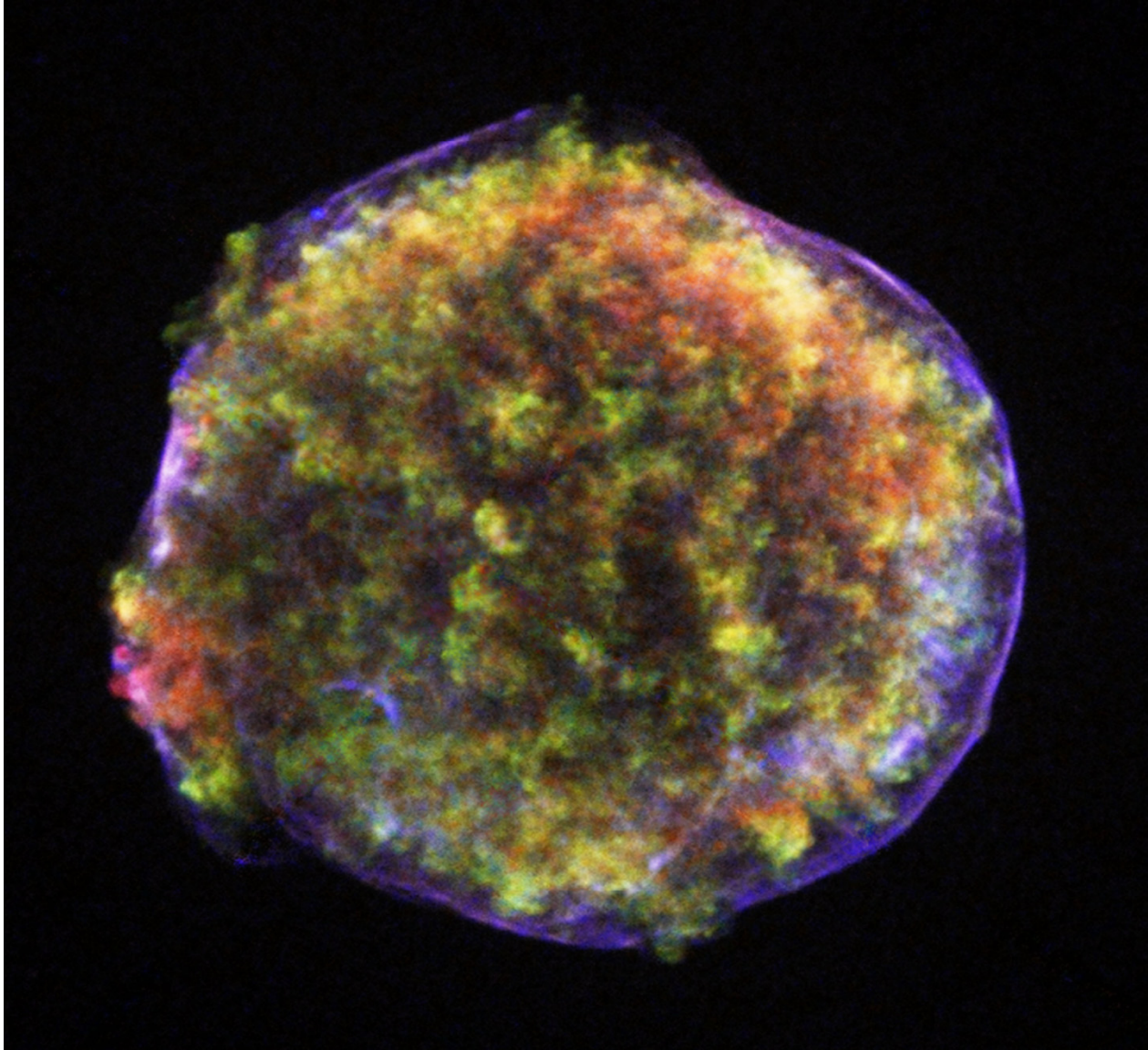
On the jitter radiation

Synchrotron radiation

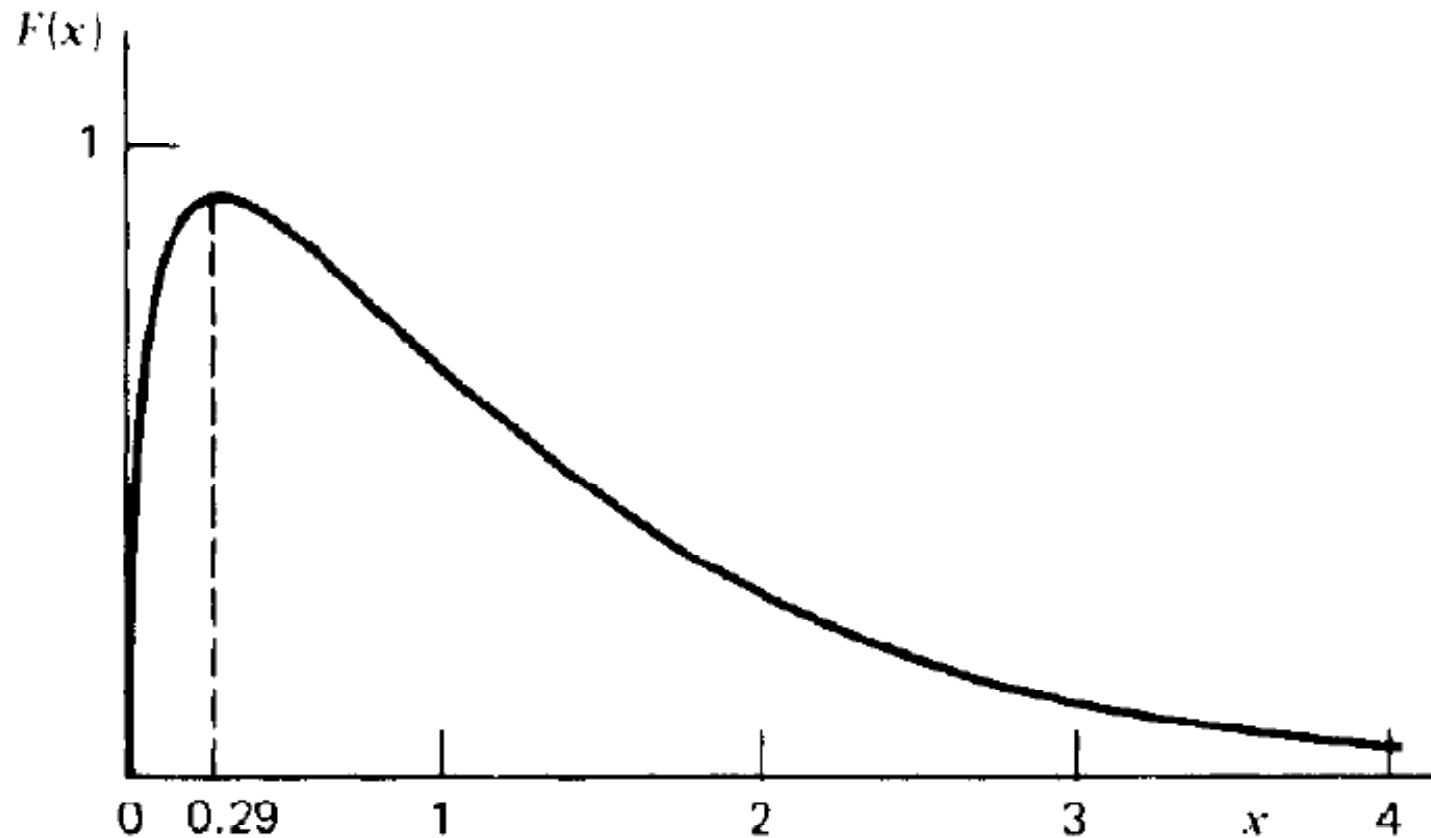
- The ultra-relativistic electron in the magnetic field



Tycho SNR – SN1572 – 2.4 kpc (7.8 kyr)



Spectrum of one synchrotron electron

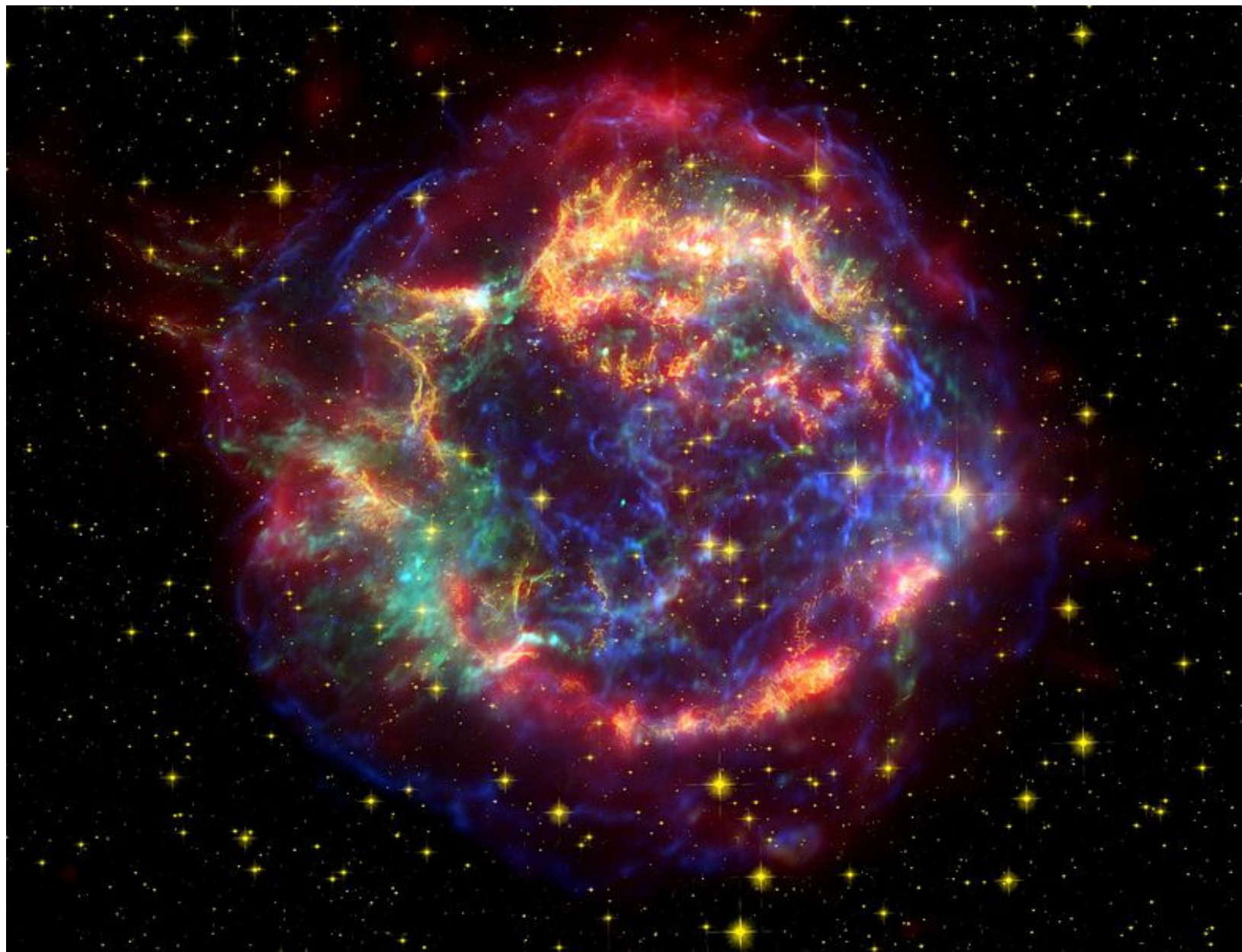


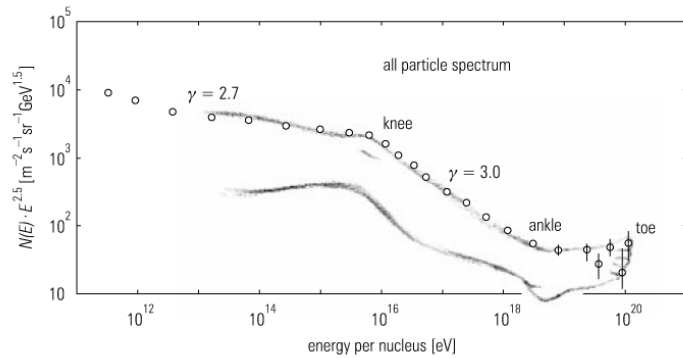
$$\nu_{\text{crit}} = c_1 E^2 B_{\perp} = 16.08 \text{MHz} (E/\text{GeV})^2 (B_{\perp}/\mu\text{G})$$

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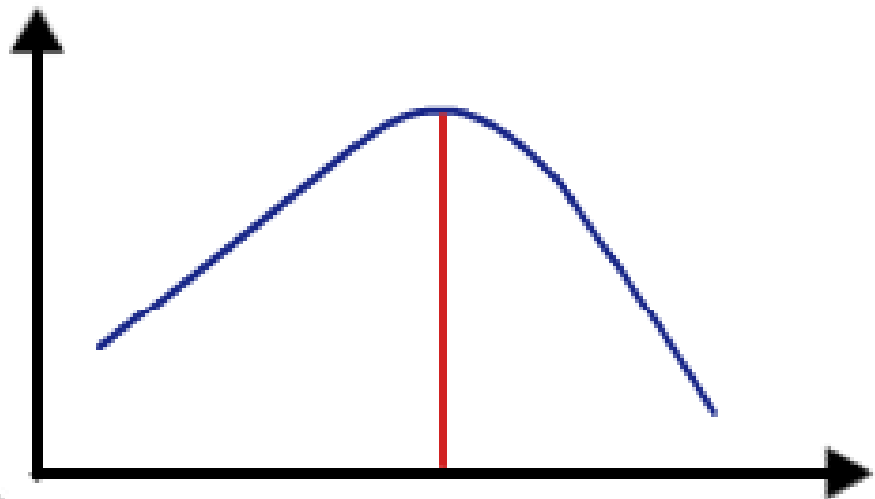
- the limiting frequencies of the radio domain,
10 MHz – 100 GHz
- the corresponding energies of electrons:
800 MeV (at 10 MHz)
80 GeV (at 100 GHz)
for $B = 1 \mu\text{G}$ - the characteristic value of ISM magnetic field.
- 800 MeV - at 1 GHz for $B = 100 \mu\text{G}$ (young SNR)
- electron energies $\sim 10 \text{ TeV} \rightarrow$ X-ray synchrotron emission
(1 keV – $2.42 \times 10^{17} \text{ Hz}$ – 1.24 nm)

Cas A – end of XVII века – 3.3 kpc (10.8 kly)





$\log_{10} F$



$$\gamma = 2a + 1$$

$\log_{10} F$

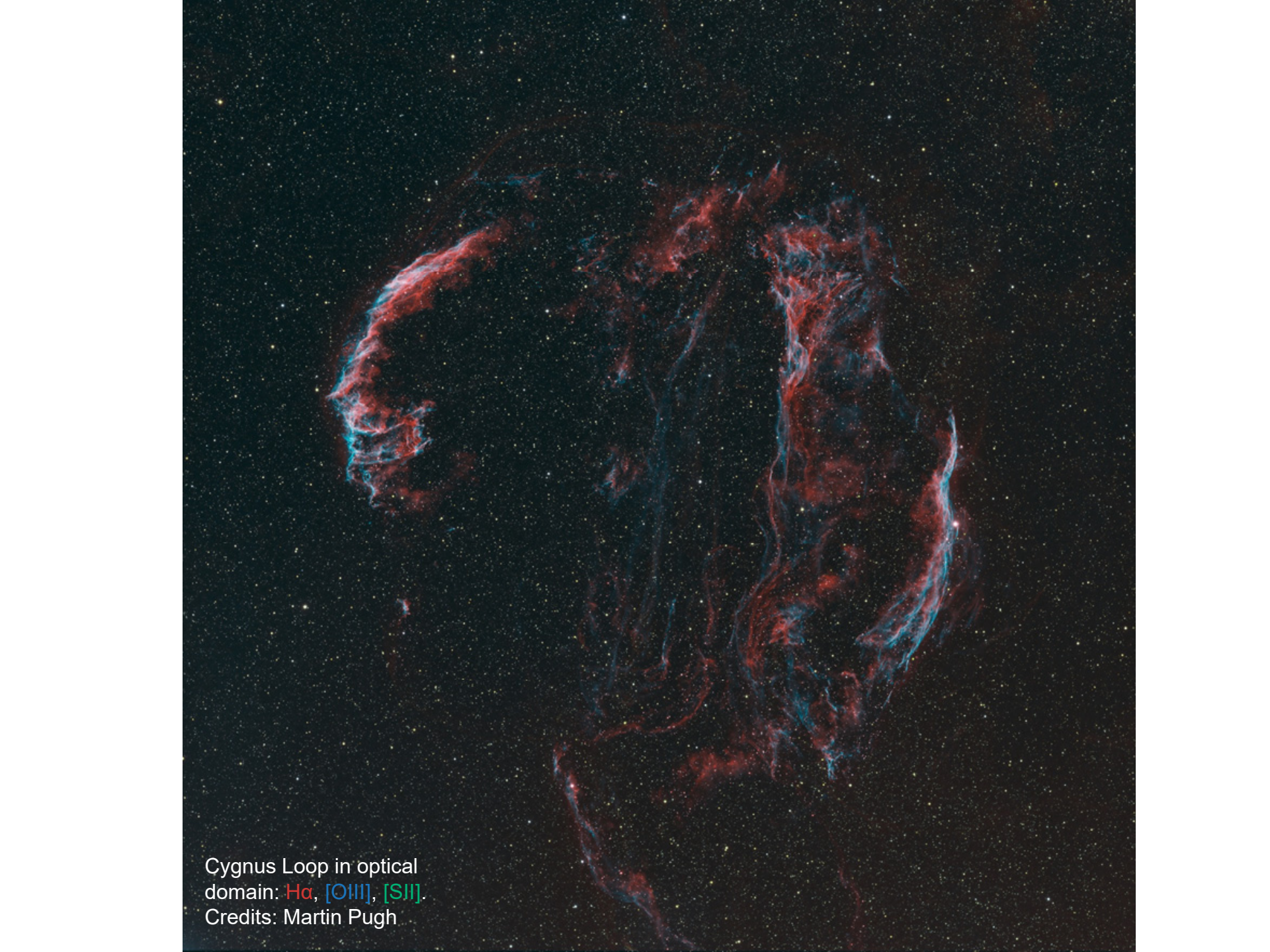


Sum of invidual contributions

$$F \sim V^a$$

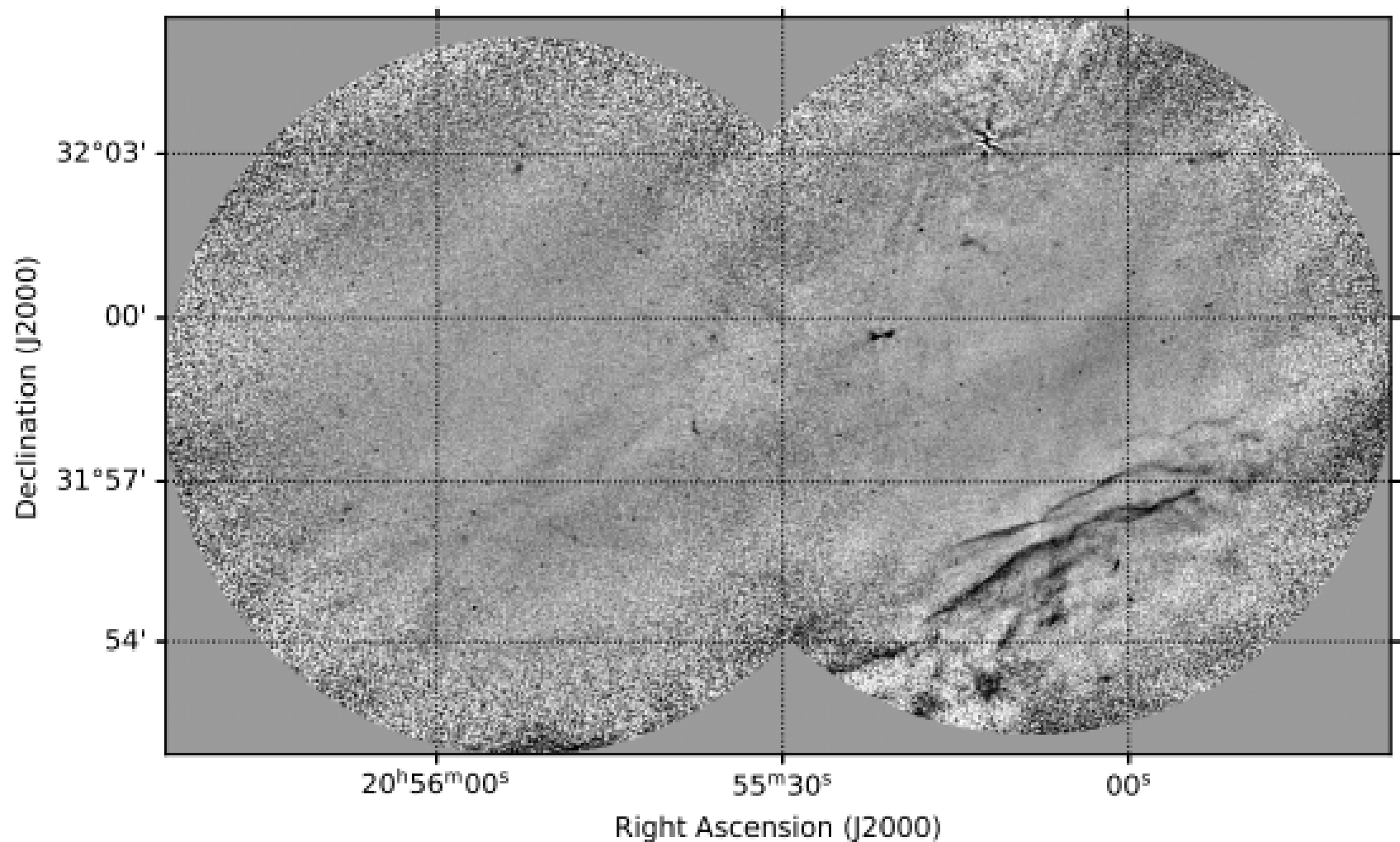
contributions from invdividual electrons

$\log_{10} V$



Cygnus Loop in optical
domain: $H\alpha$, $[O III]$, $[S II]$.
Credits: Martin Pugh

VLA – 5 GHz



Turbulences

$E(k) \sim k^{-\nu}$ – energy spectrum of turbulences

$(1/2) u_i^2 = \int E(k) dk$ - the mean turbulence kinetic energy
per unit mass)

k - the spatial frequency of eddies (wavenumber)

λ - physical size of the eddy ($k = 2\pi / \lambda$)

$\nu = 5/3$, for Kolmogorov type of turbulences

Toptygin & Fleishman (1987)

(A Role of Cosmic Rays in Generation of Radio and Optical Radiation by Plasma Mechanisms)

- Fundamental parameters:

- Larmor radius $r = \frac{E}{eB} = 3.33 \times 10^{12} \frac{(E/\text{GeV})}{(B/\mu\text{G})} \text{ cm.}$

- Larmor frequency $\omega_B = eB/mc,$

- Plasma frequency $\frac{\nu_p}{\text{kHz}} = 8.97 \sqrt{\frac{N_e}{\text{cm}^{-3}}},$

- Critical (synchrotron) frequency $\omega_c = \omega_B \gamma^2,$

Jitter radiation of one electron

Low frequency $\omega_p \ll \omega \ll \omega_p \gamma$

- Random mag. fields:

$$I_\omega \approx \frac{2^{\nu+3} \Gamma(\nu/2)}{9 \sqrt{\pi} \Gamma(\nu/2 - \frac{1}{2})} \frac{e^2 \omega_{st}^2}{c \gamma^2 \omega_p} \left(\frac{\omega_*}{\omega_p} \right)^{\nu-1} \left(\frac{\omega}{\omega_p} \right)^{\nu+2}$$

- Regular mag. fields ($\Omega_\perp \gamma \gg \omega_{st}$)

$$I_\omega \approx \frac{e^2}{2 \sqrt{\pi} c} (\Omega_\perp \omega_p)^{1/2} \exp(-2 \omega_p^3 / 3 \Omega_\perp \omega^2)$$

Radiation is strongly affected by the density effect (theory of bremsstrahlung and magnetic bremsstrahlung in regular magnetic field)

Jitter radiation of one electron

Intermediate frequencies $\omega_p \gamma \ll \omega \ll \Omega_{\perp} \gamma^3$

- Large-scale field is sufficiently small ($\omega_{st} \gg \Omega_{\perp} \gamma, \omega_* > \omega_{st}$):

$$I_{\omega} \approx \left[\frac{4\Gamma(v/2)}{3\pi^{3/2}\Gamma(v/2 - \frac{1}{2})} \right]^{1/2} \frac{e^2 \omega_{st}}{c} \left(\frac{\omega}{\omega_* \gamma^2} \right)^{1/2}$$

- Large-scale field is strong enough ($\Omega_{\perp} \gamma \gg \omega_{st}$):

$$I_{\omega} \approx \frac{3^{1/6}\Gamma(2/3)}{\pi} \frac{e^2}{c\gamma^2} (\Omega_{\perp} \gamma^3)^{2/3} \omega^{1/3}$$

Random fields play little role – synchrotron emission!!!

Jitter radiation of one electron

High frequencies $\omega \gg \Omega_{\perp} \gamma^3$

- Up to $\omega_{\max}$ for the minimal turbulence scale ($\omega_{\max} = c \gamma^2 / l_{\min}$):

$$I_{\omega} \approx \frac{2^{\nu+1} \Gamma(\nu/2)}{3 \pi^{1/2} (\nu + 2) \Gamma(\nu/2 - \frac{1}{2})} \frac{e^2}{c} \omega_{st}^2 \gamma^2 (\omega_* \gamma^2)^{\nu-1} \omega^{-\nu}$$

- If turbulence spectrum is not power-law:

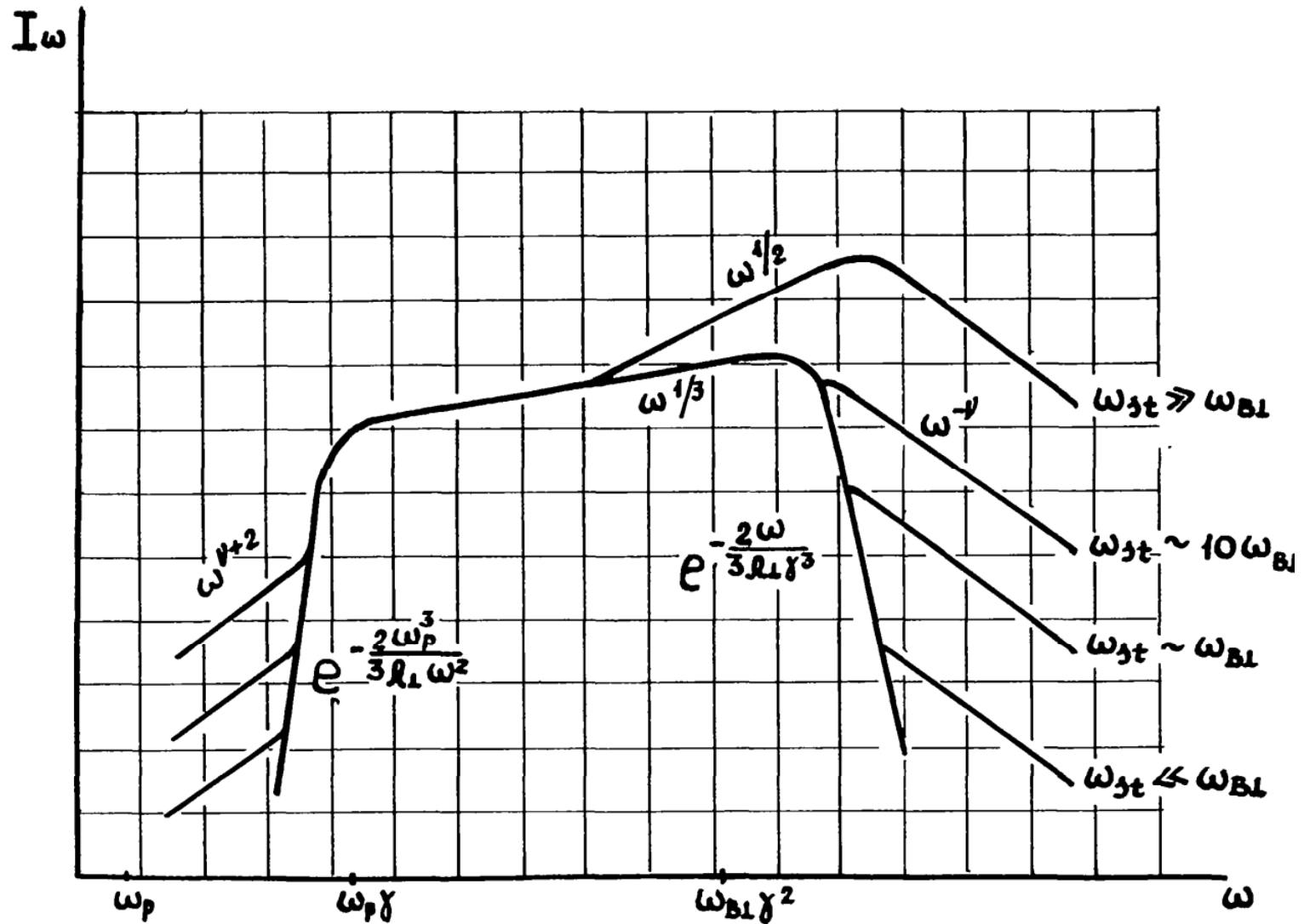
$$I_{\omega} \approx \frac{8 e^2 q'(\omega)}{3 \pi c} \gamma^2$$

- Absence of random inhomogeneities – exponential fall:

$$I_{\omega} \approx \frac{e^2}{2 \sqrt{\pi} c \gamma^2} (\Omega_{\perp} \gamma^3 \omega)^{1/2} \exp(-2 \omega / 3 \Omega_{\perp} \gamma^3)$$

Again synchrotron emission!!!

Jitter radiation of one electron



Jitter radiation of an ensemble of electrons

High frequencies $\omega \gg \omega_* = \omega_{B\perp} \gamma_2^2 \Lambda, \quad \gamma_1 \leq \gamma \leq \gamma_2$

- Λ depends on relative level of the turbulence:

$$\Lambda = \frac{3}{2} \ln \left[\frac{3^{\nu+3/2} (\nu+2) \Gamma(\nu/2 - 1/2) \omega_{B\perp}^{\nu+1}}{2^{2\nu+5/2} \Gamma(\nu/2) \omega_{st}^2 \omega_0^{\nu-1}} \right]$$

$$P(\omega) = \int_{\gamma_1}^{\gamma_2} I_\omega(\gamma) dN_e(\gamma)$$

- The radiation spectrum repeats the turbulence spectrum:

$$P_1(\omega) = \frac{2^{\nu+1} \Gamma(\nu/2) K_e \gamma_2^{2\nu+1-\xi}}{3 \pi^{1/2} (\nu+2) (2\nu+1-\xi) \Gamma(\nu/2 - \frac{1}{2})} \frac{e^2}{c} \omega_{st}^2 \omega_0^{\nu-1} \omega^{-\nu}$$

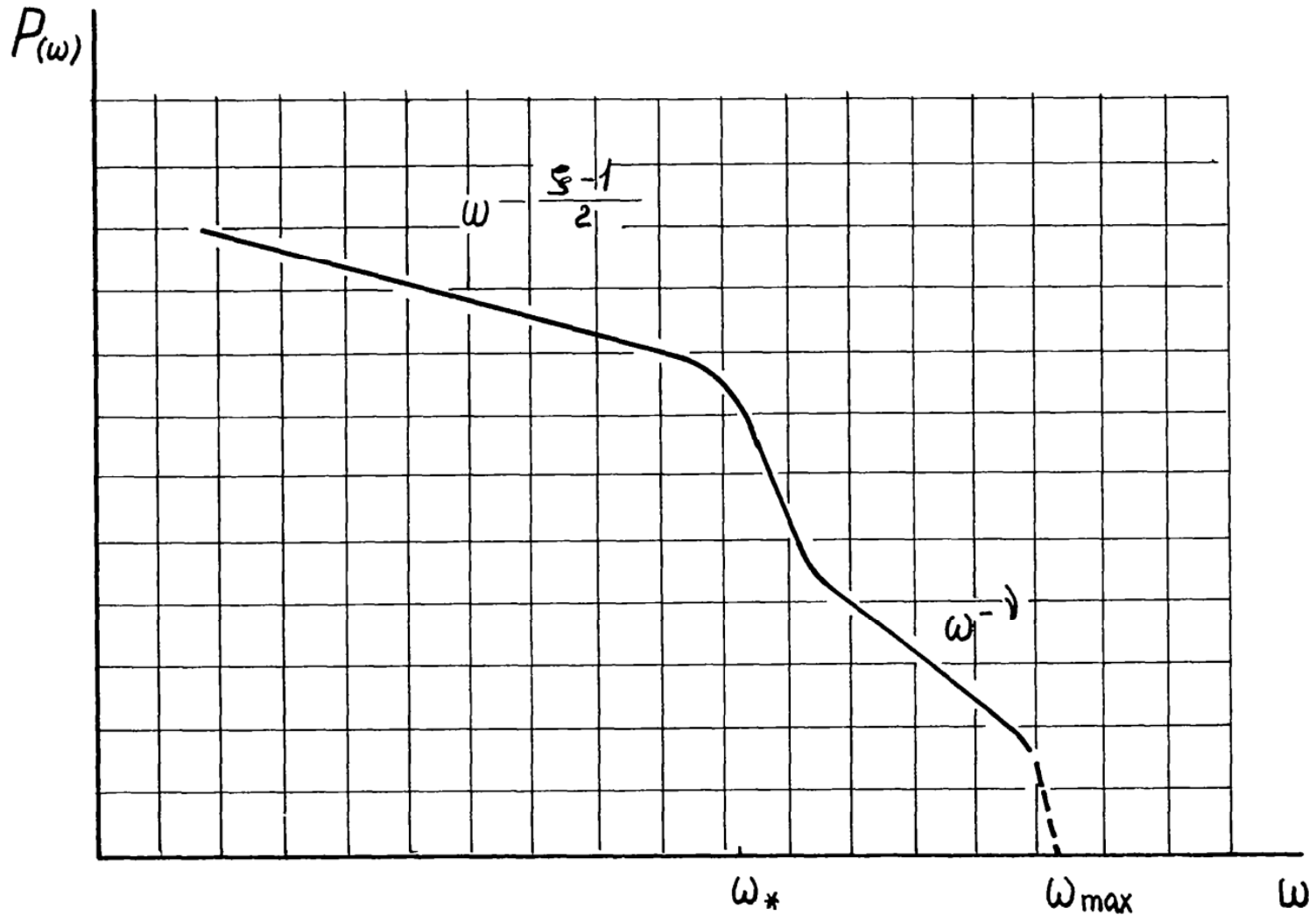
Jitter radiation of an ensemble of electrons

Intermediate frequencies $\omega_{**} \ll \omega \ll \omega_*$, $\omega_{**} \approx \omega_p(\omega_p \gamma_1 / \omega_{B\perp})^{1/2}$, at $\omega_{B\perp} \gg \omega_{st}$:

$$P_2(\omega) = \frac{3^{\xi/2} \Gamma(\xi/4 - \frac{1}{12}) \Gamma(\xi/4 - \frac{19}{12}) K_e}{\xi + 1} \frac{e^2}{c} \omega_{B\perp}^{\alpha+1} \omega^{-\alpha}$$

- If the small-scale field is dominant and $\omega_{st} \gg \omega_{B\perp}$ is obtained again power-law with the same spectral index!!!
- Numerical factor cannot be found analytically, and requires numerical calculations!

Jitter radiation of an ensemble of electrons



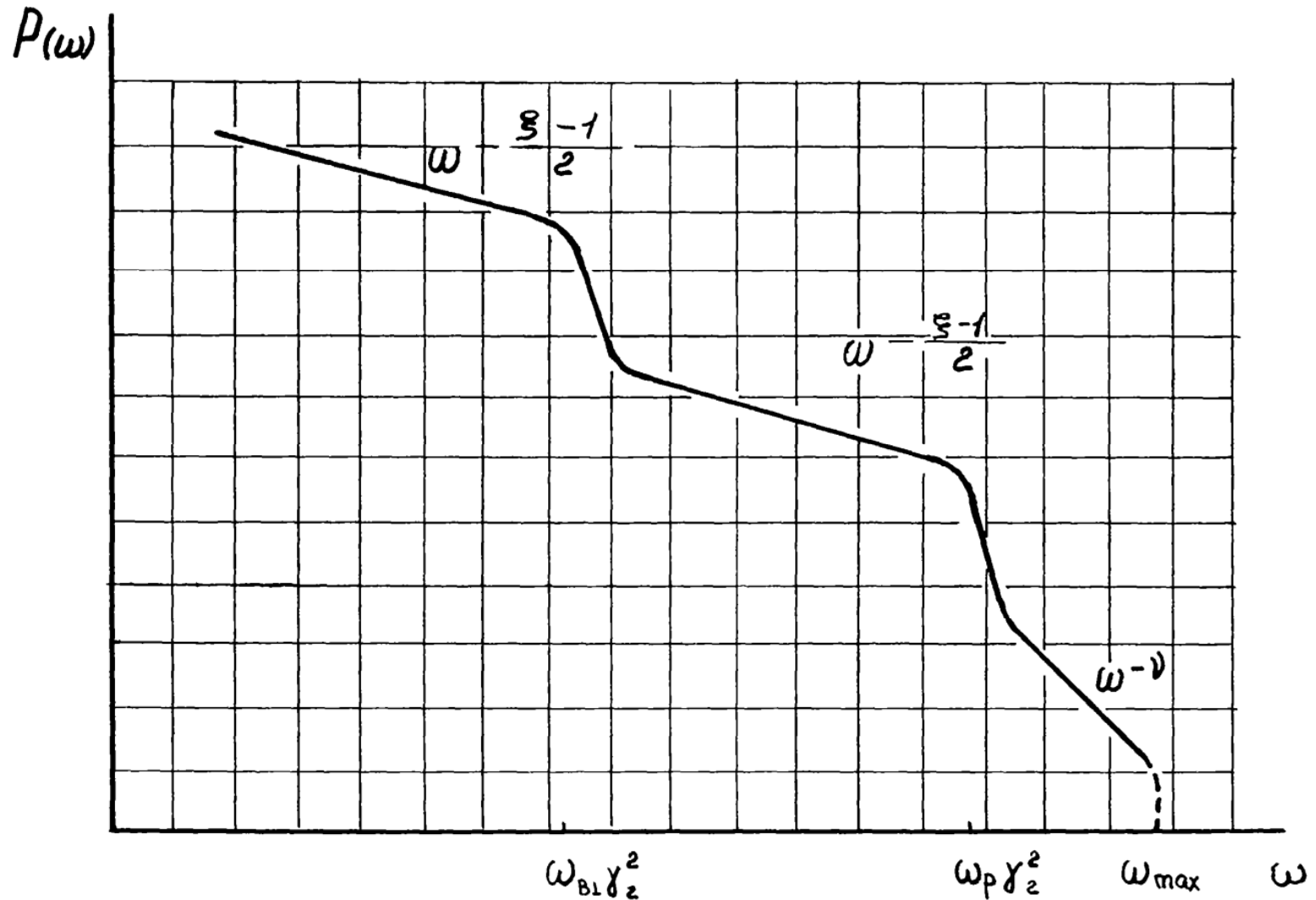
Jitter radiation of an ensemble of electrons

Intermediate frequencies – with adding of Langmuir small-scale turbulences where $\omega_E^2 = e^2 \langle E^2 \rangle / m^2 c^2$:

$$P_l(\omega) = \frac{2^{\xi/2 - 3/2} (\xi^2 + 4\xi + 11)}{(\xi + 1)(\xi + 3)(\xi + 5)} K_e \frac{e^2}{c} \frac{\omega_E^2}{\omega_p} \left(\frac{\omega_p}{\omega} \right)^\alpha, \quad \alpha = \frac{\xi - 1}{2},$$

for $\omega_{B\perp} \gamma_2^2 < \omega < \omega_p \gamma_2^2$

Jitter radiation of an ensemble of electrons



Jitter radiation of an ensemble of electrons

Low frequencies $\omega \lesssim \omega_{**}$; follow intrinsic radiation of one relativistic electron:

$$P_3(\omega) = \frac{2^{\nu+5} \Gamma(\nu/2) K_e}{9 \pi^{1/2} (\xi + 1) \Gamma(\nu/2 - \frac{1}{2})} \frac{e^2}{c} \frac{\omega_{st}^2 \omega_0^{\nu-1}}{\omega_p^\nu \gamma_1^{\xi-1}} \left(\frac{\omega}{\omega_p} \right)^{\nu+2}$$

Transition radiation – intrinsic radiation is suppressed by the density effects – significant influence of CR nuclei:

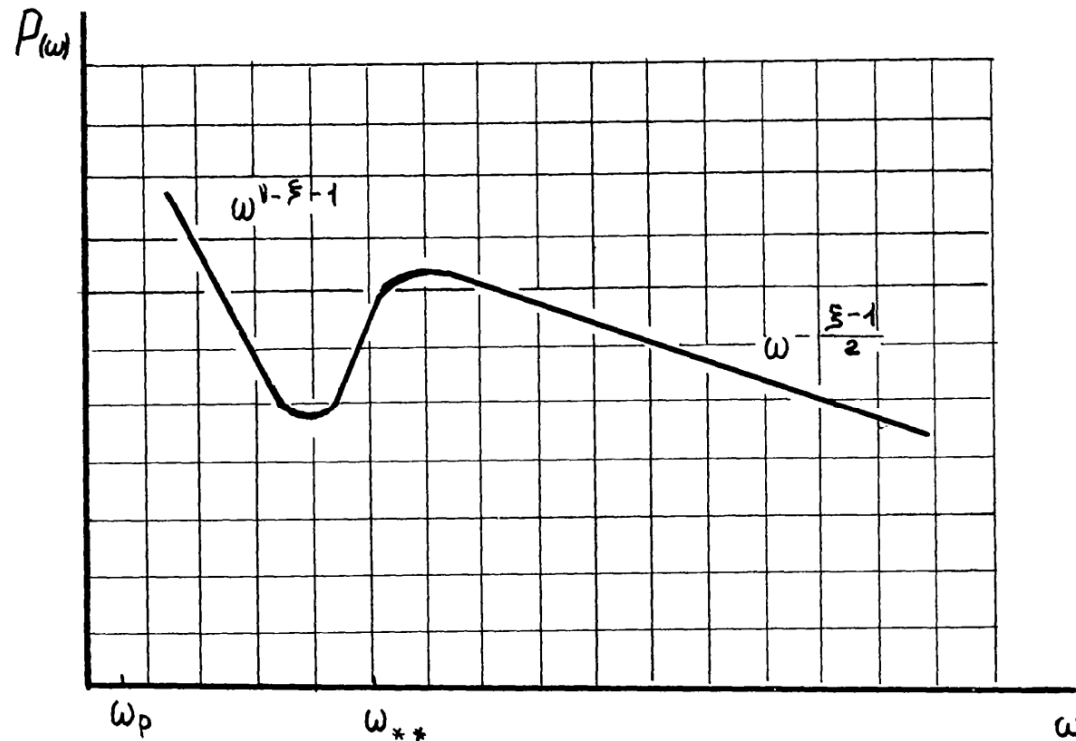
$$P^m(\omega) = \frac{2^{\nu-2} (\nu - 1) \Gamma(\xi/2 + \frac{1}{2}) \Gamma(\nu - \xi/2 + \frac{1}{2}) K_{cr}}{\nu (\xi - 1) \Gamma(\nu + 2)} \times$$

$$\times \frac{q^2 \omega_{st}^2}{c \omega_p} \left(\frac{\omega_0}{\omega_p} \right)^{\nu-1} \left(\frac{\omega_p}{\omega_B} \right)^2 \left(\frac{\omega_p}{\omega} \right)^{\xi+1-\nu}.$$

Jitter radiation of an ensemble of electrons

Low frequencies $\omega \lesssim \omega_{**}$:

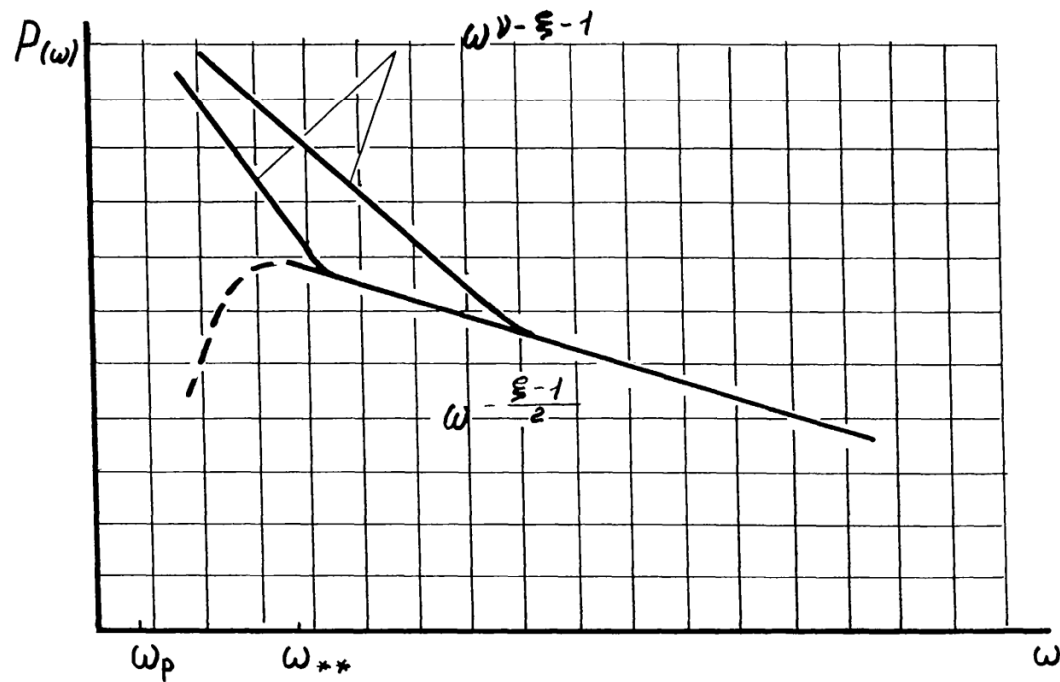
$$P(\omega) = P_3(\omega) + P^m(\omega)$$



Jitter radiation of an ensemble of electrons

Low frequencies $\omega \lesssim \omega_{**}$; if density effects are absent the transition radiation has a form:

$$P^a(\omega) = \frac{(\nu - 1)\Gamma(\xi/2 + \frac{1}{2})\Gamma(\nu - \xi/2 + \frac{1}{2})K_{cr}}{\nu(\xi - 1)\Gamma(\nu + 2)} \frac{q^2\omega_{st}^2}{c\omega_p} \left(\frac{2\omega_0}{\omega_p}\right)^{\nu-1} \left(\frac{\omega_p}{\omega}\right)^{\xi+3-\nu}$$



Jitter radiation of an ensemble of electrons

Low frequencies $\omega \gg \omega_p$; transition radiation produced by a small density jumps (induced by an ensemble of weak shocks):

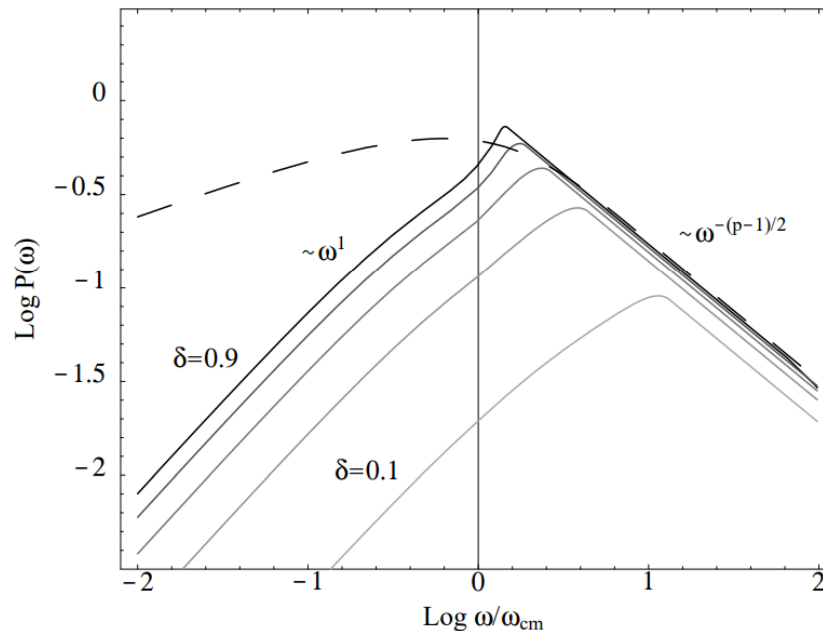
$$P^{tr}(\omega) = \frac{q^2 K_{cr}}{12 \pi L} \Gamma(\xi/2 - \frac{1}{2}) \Gamma(\frac{5}{2} - \xi/2) \left(\frac{\Delta N}{N_0} \right)^2 \left(\frac{\omega_p}{\omega} \right)^{\xi - 1}$$

The spectrum of the transition radiation is steeper than the synchrotron spectrum and gives larger contribution into the low-frequency range!

Medvedev (2000)

(Theory of “Jitter” Radiation from Small-Scale Random Magnetic Fields and Prompt Emission of Gamma-Ray Burst Shocks)

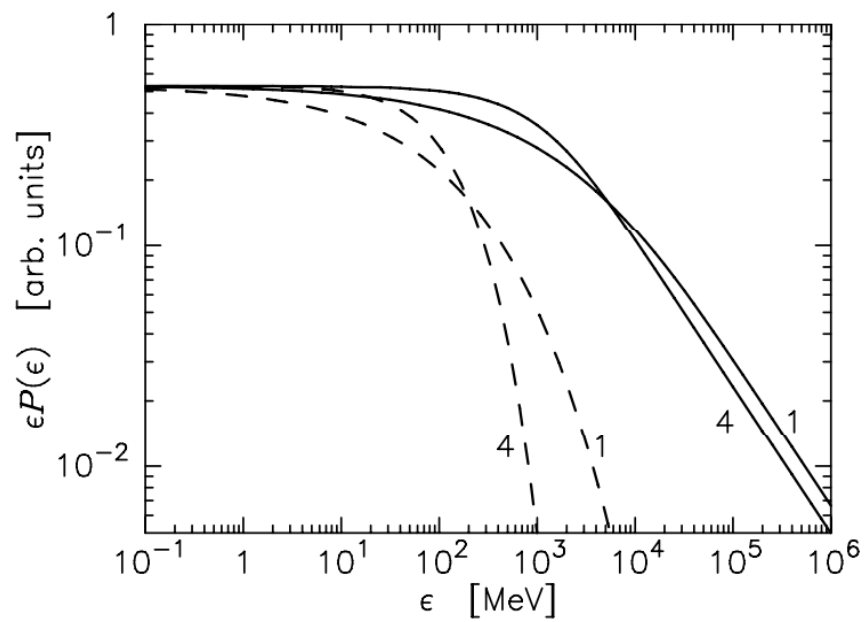
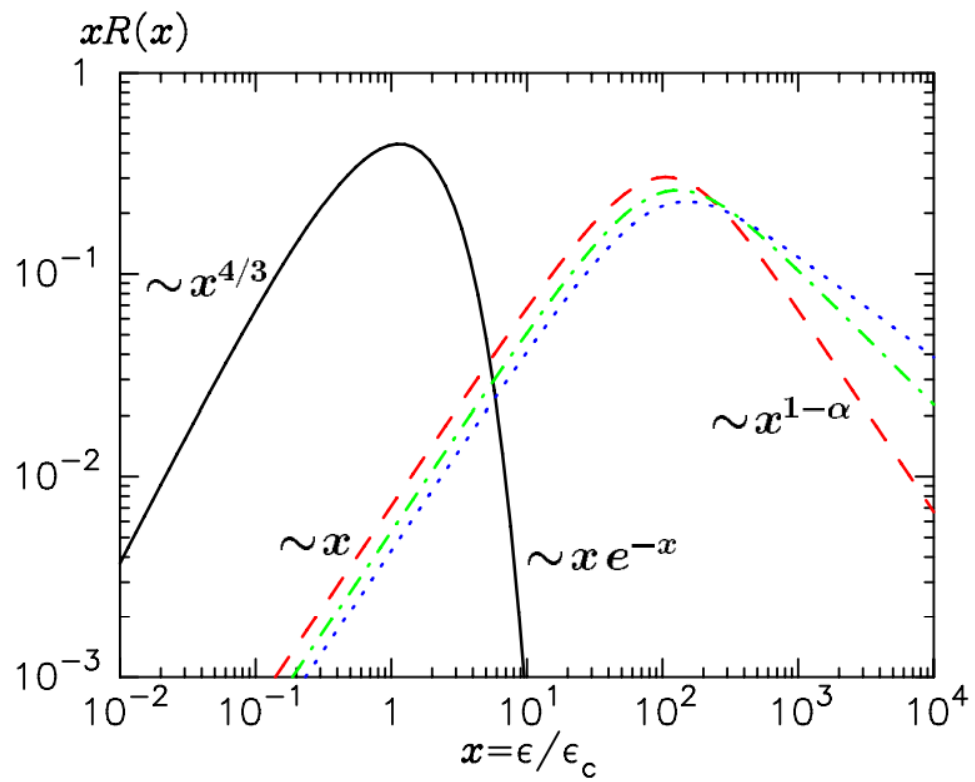
- Small-scale fields generated only by the electrons → jitter radiation
- Larger scale fields generated only by the protons → synchrotron radiation
- Peak frequency of jitter radiation is independent of the magnetic field strength – it depends on density ($\omega_{jm} \sim n^{1/2}$)



Kelner, Aharonian & Khangulyan (2013)

(On the Jitter Radiation)

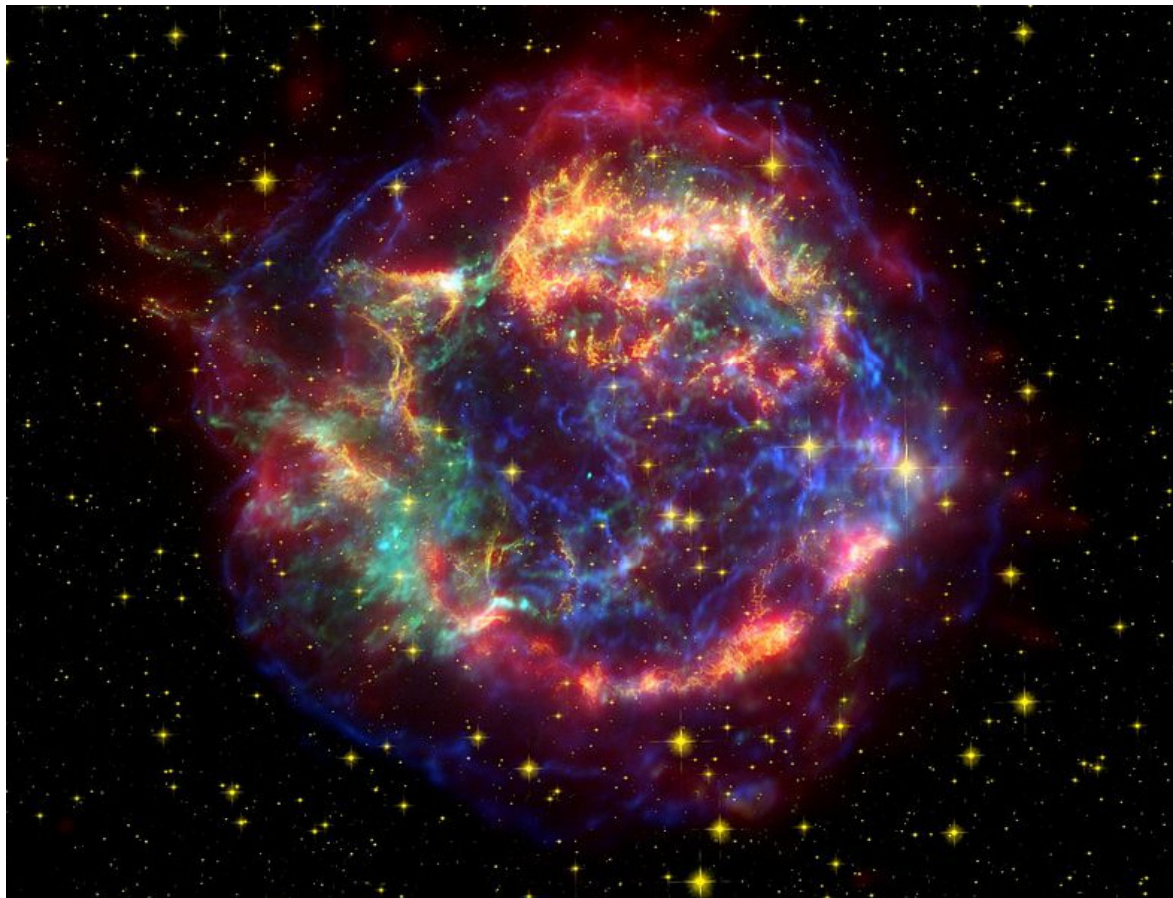
- Magnetic bremsstrahlung in small-scale turbulent plasmas → jitter radiation
- Shifted toward higher energy (gamma-rays) compared to synchrotron radiation
- Attractive broad-band gamma-ray production channel
- Framework is perturbation theory on TF87 theory

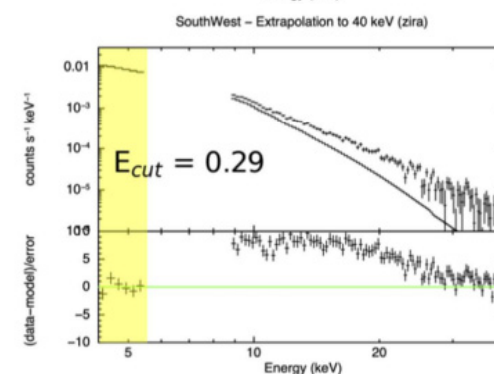
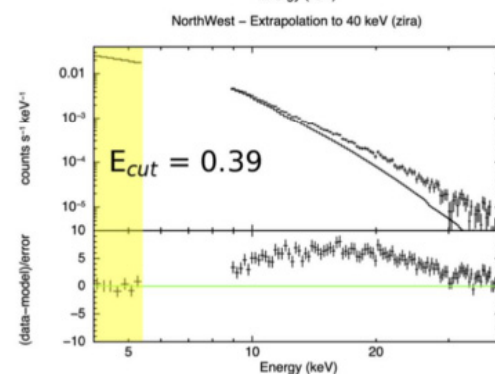
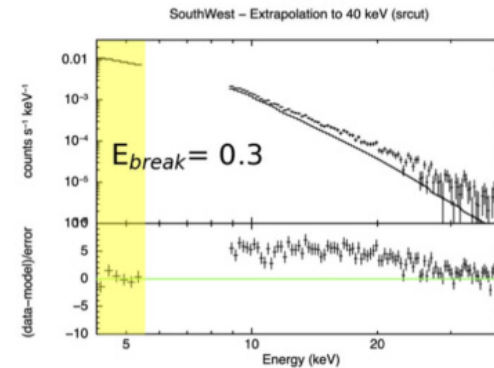
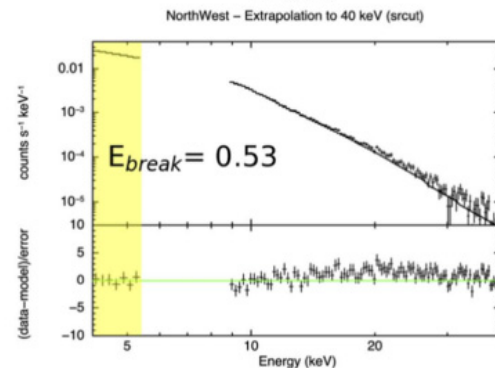
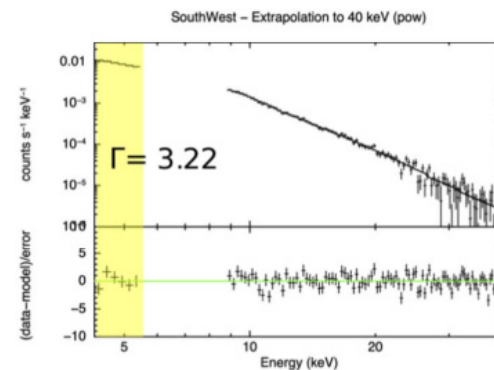
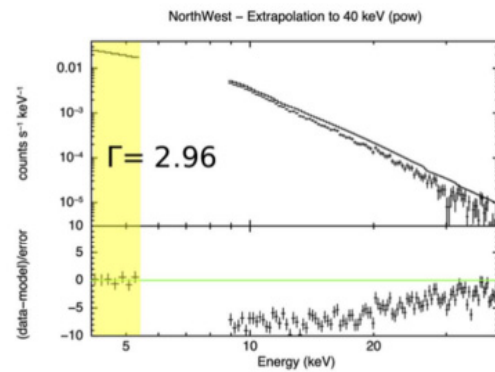
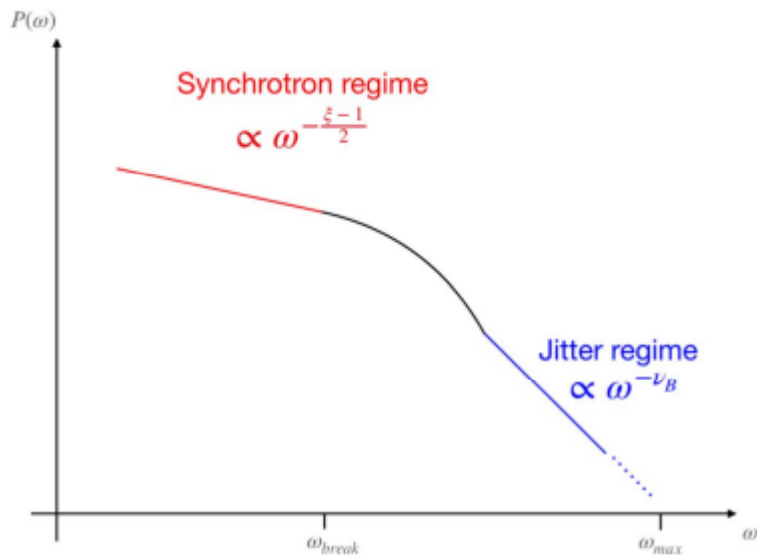


- Small-scale turbulences $\rightarrow \lambda \ll 1.7 \times 10^3 (m/m_e) (B/1\text{G})^{-1} \text{ cm}$
- Condition of the jitter regime does not depend on the particle energy but only on its mass!
- Turbulence scale as small as:
 - 100 km in young SNRs (100 μG)
 - less than 10 m in the gamma-ray production regions of blazars (1 G)
 - 1 cm in GRBs (1 kG)
- If the ratio $R_L / \lambda > 10 \rightarrow$ power-law tail of the jitter radiation is extended more than two order of magnitude in energies after the maximum!!!

Greco, Vink, Ellien & Ferrigno (2023)

(Jitter Radiation as an Alternative Mechanism for the
Nonthermal X-Ray Emission of Cassiopeia A)





- DSA intrinsically requires high magnetic turbulence – the spectral models used to fit nonthermal spectra of SNRs do not include the effect of turbulence!
- The 15 -100 keV spectra of Cas A fitted power-law photon index of $\Gamma \sim 3.3$ (4 - 40 keV, jitter model is better than exp. cutoff model: $\Gamma \sim 3 - 3.4$)
- Steepening of the spectrum is incompatible with the exponential cutoff!
- $\Gamma \sim 3.3 \rightarrow$ the spectral index of turbulent spectrum $\nu \sim 2.3$!
 Kolmogorov turbulence: $\nu \sim 5/3$
 Kraichnan turbulence: $\nu \sim 3/2$
 Takabe (2023), Weibel instabilities : $\nu \sim 2$
- Turbulence scale is typically smaller than 100 km!
- Low polarization $\sim 3\% \rightarrow$ nonthermal X-ray emission is mix of synchrotron and jitter components!!!



THANK YOU VERY MUCH FOR
YOUR ATTENTION!!!