Teorema Nehoroševa i asteroidni prsten kao dinamički sistem

R. Pavlović

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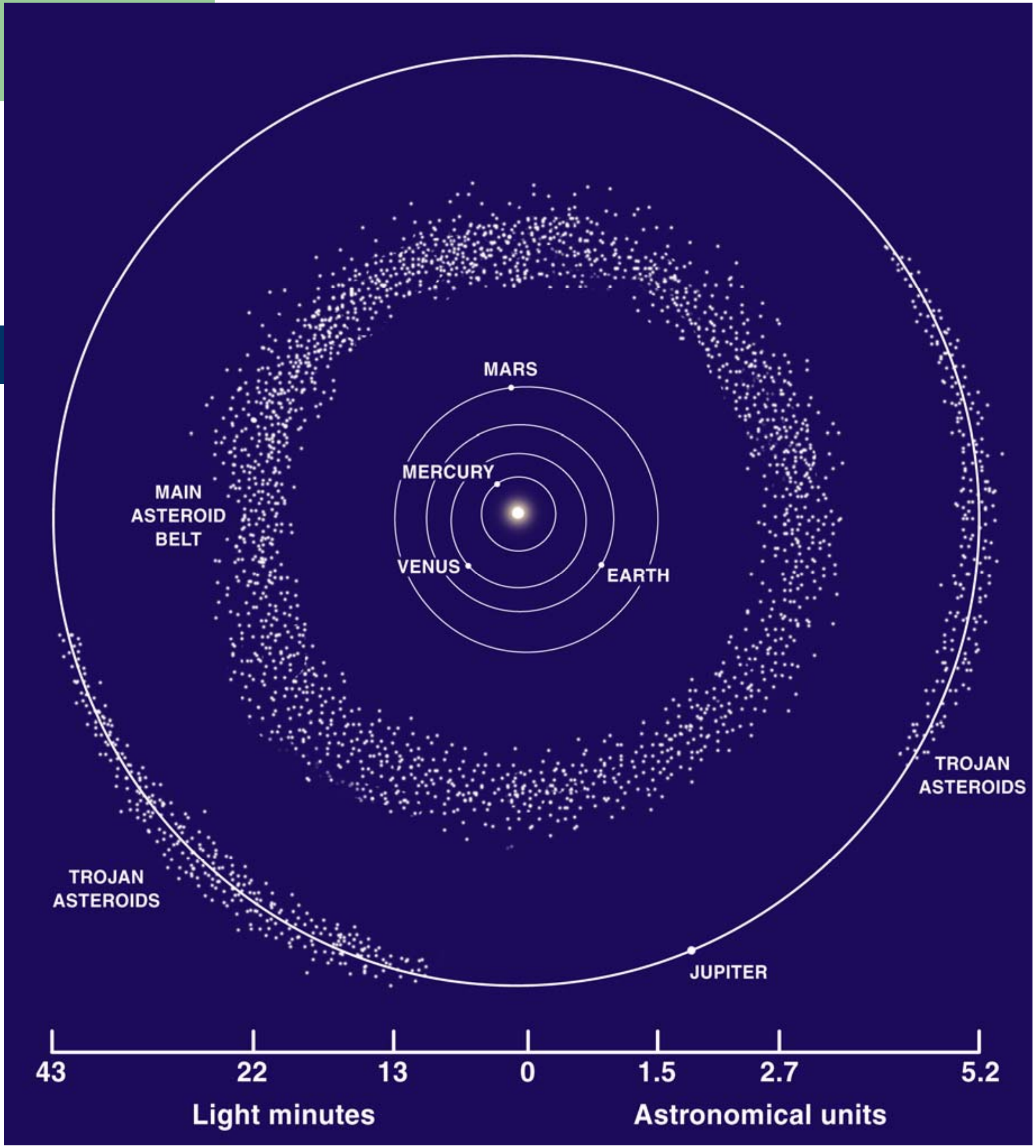
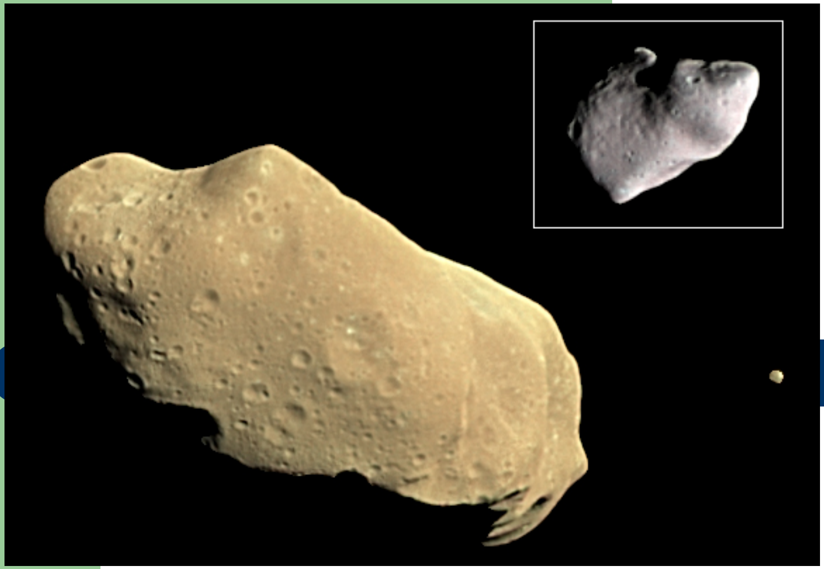
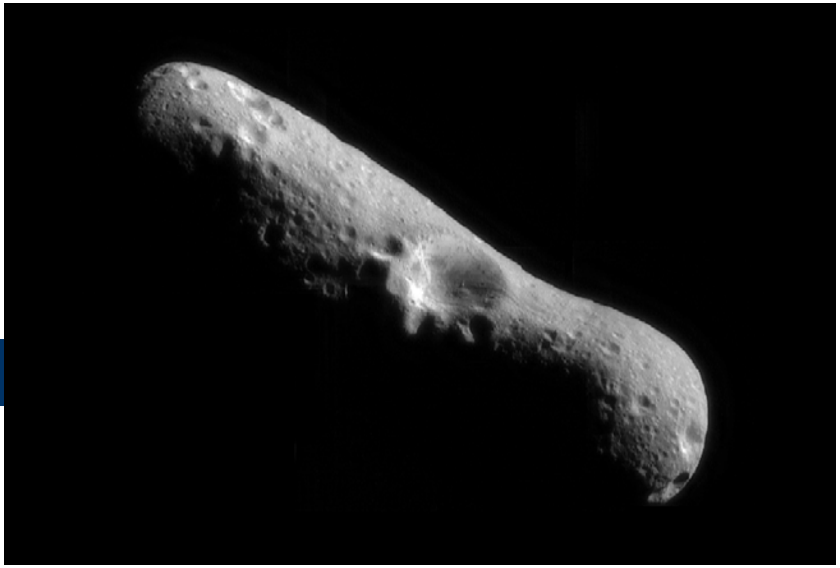
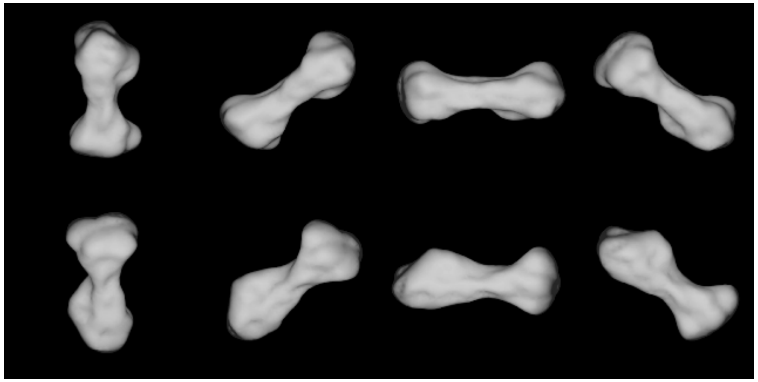

MARS
MERCURY
VENUS
EARTH
MAIN ASTEROID BELT
TROJAN ASTEROIDS
TROJAN ASTEROIDS
JUPITER
43
22
13
0
1.5
2.7
5.2
Light minutes
Astronomical units



Ida i Dactil



Eros



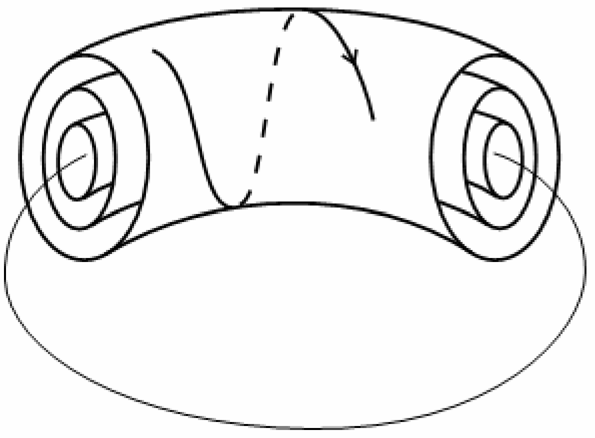
Kleopatra

Pitanje: da li je asteroidni pojas

- Veliko “haotično more” – ostatak mnogo veće populacije
- Poseduje strukturu Nehoroševa – neka vrsta “permanentne” konfiguracije

Teoreme

- **KAM**
- **Nehoroševa**



- *Kvazi-integrabilni*
- *Nedegenerisani*

$$h = h_0(I) + \varepsilon f(I, \phi)$$

Teorema Nehoroševa

Neka je $\mathcal{H}(p,q) = \mathcal{H}_0 + \epsilon\mathcal{H}_1$ realan i analitički u $\mathcal{D} \equiv \mathcal{G} \times \mathrm{T}^n$, gde je $\mathcal{G} \subset \mathbb{R}^n$ otvoren i ograničen i $\|\mathcal{H}_1\| \leq 1$. Neka je matrica $C(p)$ definisana sa $C_{ij} = \frac{\partial^2 \mathcal{H}_0(p)}{\partial p_i \partial p_j}$ i neka postoje pozitivne konstante M i m takve da

$$\|C(p)v\| \leq M\|v\|, \forall p \in G, \forall v \in \mathbb{R}^n \quad (1)$$

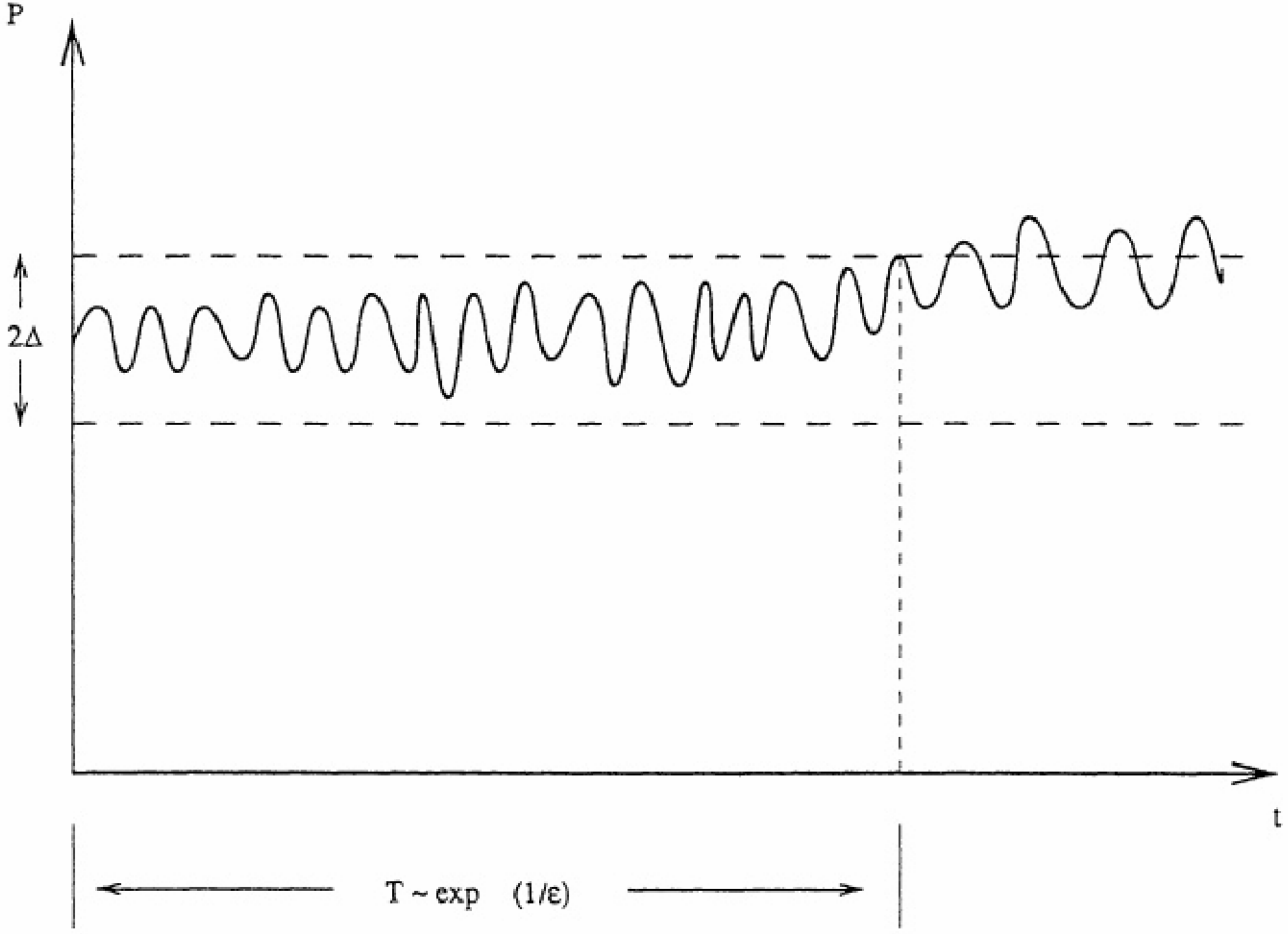
$$|C(p)v \cdot v| \geq mv \cdot v, \forall p \in G, \forall v \in \mathbb{R}^n. \quad (2)$$

Tada postoje pozitivne konstante ϵ_*,α, β, a i b takve da za svako $\epsilon \leq \epsilon_*$ važi

$$\|p(t) - p(0)\| \leq \Delta \equiv \alpha\epsilon^a \quad (3)$$

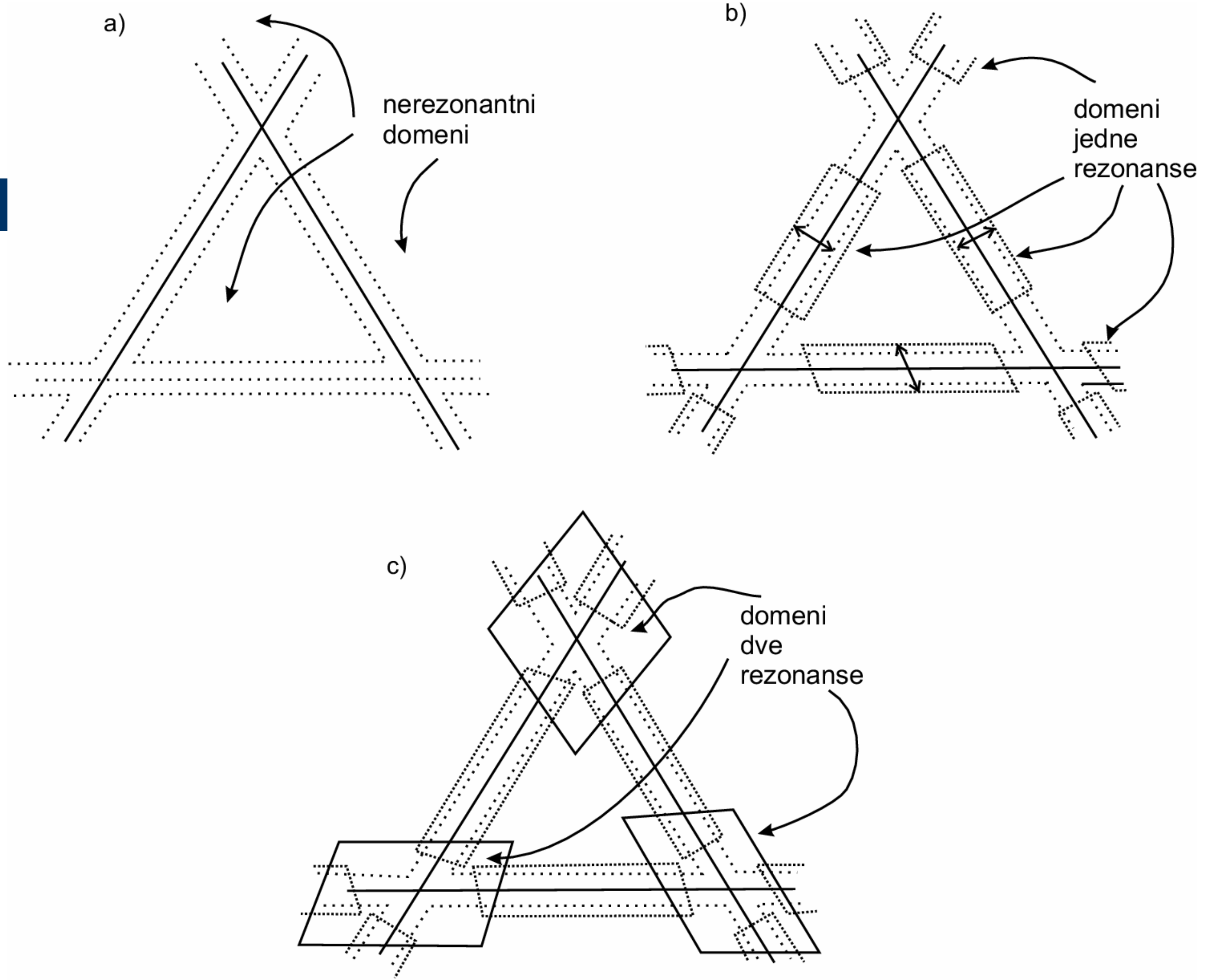
za svako $p(0) \in \mathcal{G} - \Delta$ i za svako $|t| \leq T(\epsilon)$ gde je

$$T(\epsilon) = \beta\left(\frac{1}{\epsilon}\right)\exp\left(\frac{1}{\epsilon}\right)^b. \quad (4)$$



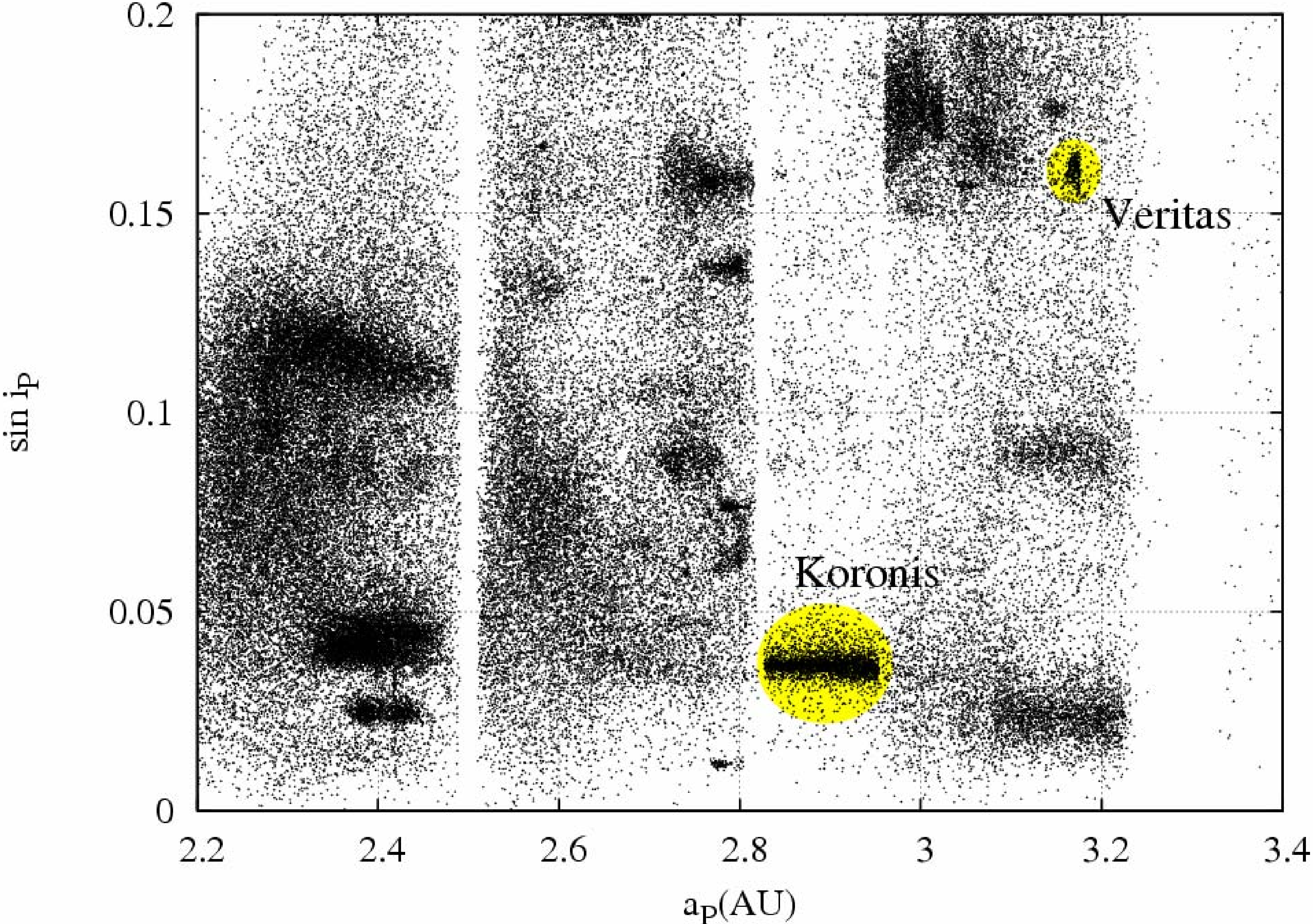
P
2Δ
t
T ~ exp (1/ε)

Geografija rezonansi



Urađeno

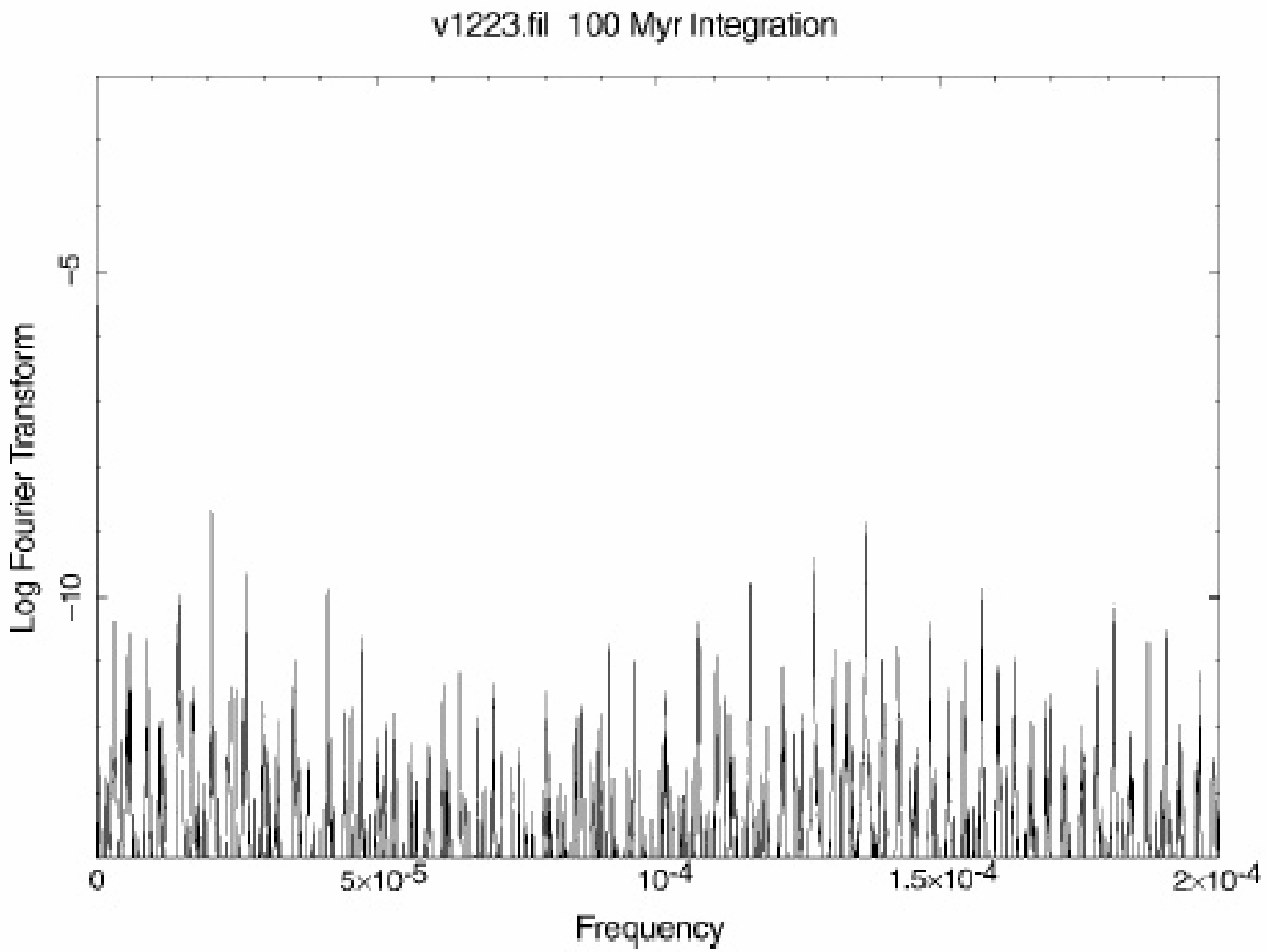
- Ispitani uslovi za primenu teoreme Nehoroševa



0.2
0.15
0.1
0.05
0
sin i_P
2.2
2.4
2.6
2.8
3
3.2
3.4
a_P(AU)
Veritas
Koronis

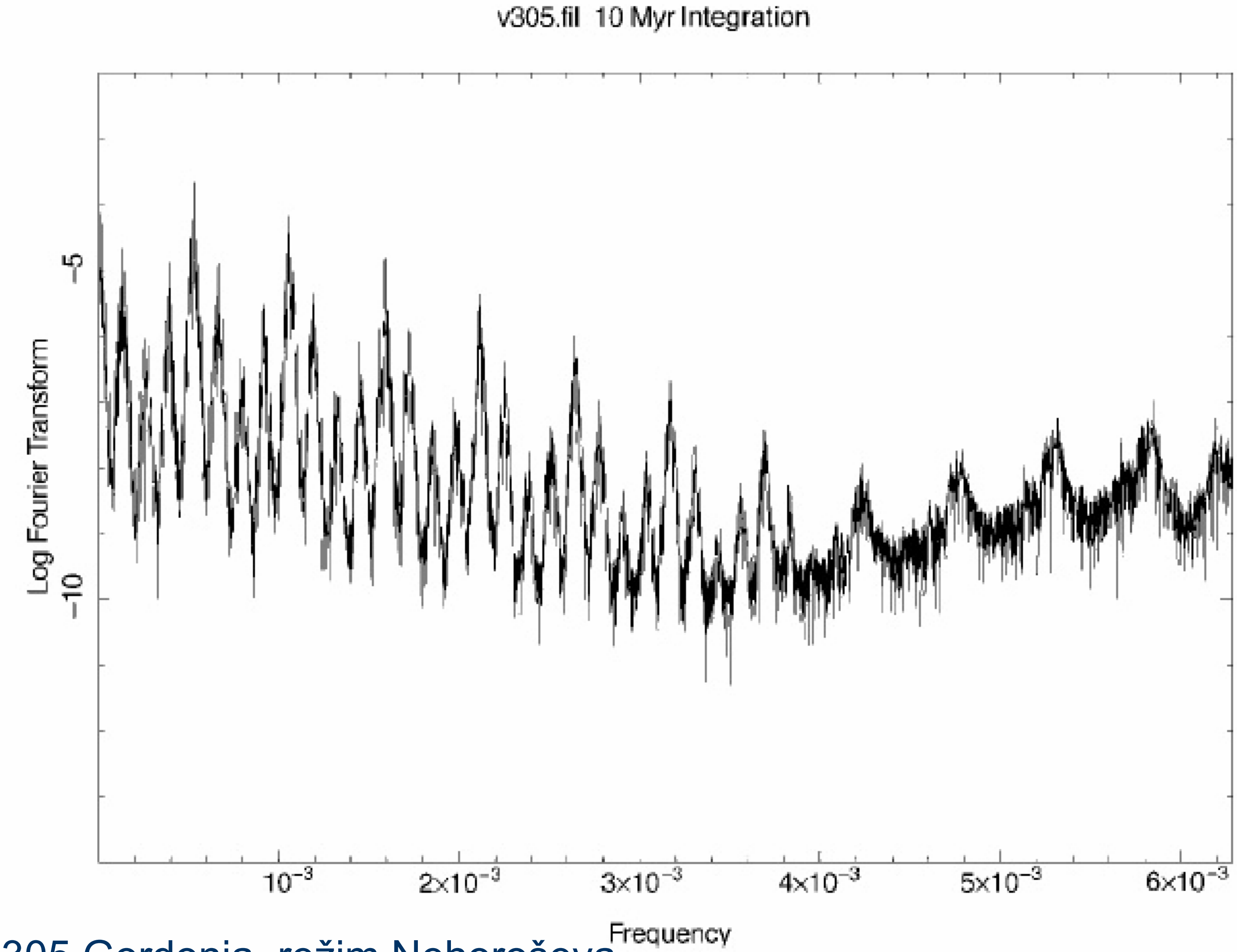
- Primenjena teorema Nehoroševa u spektralnoj formulaciji

Stabilni



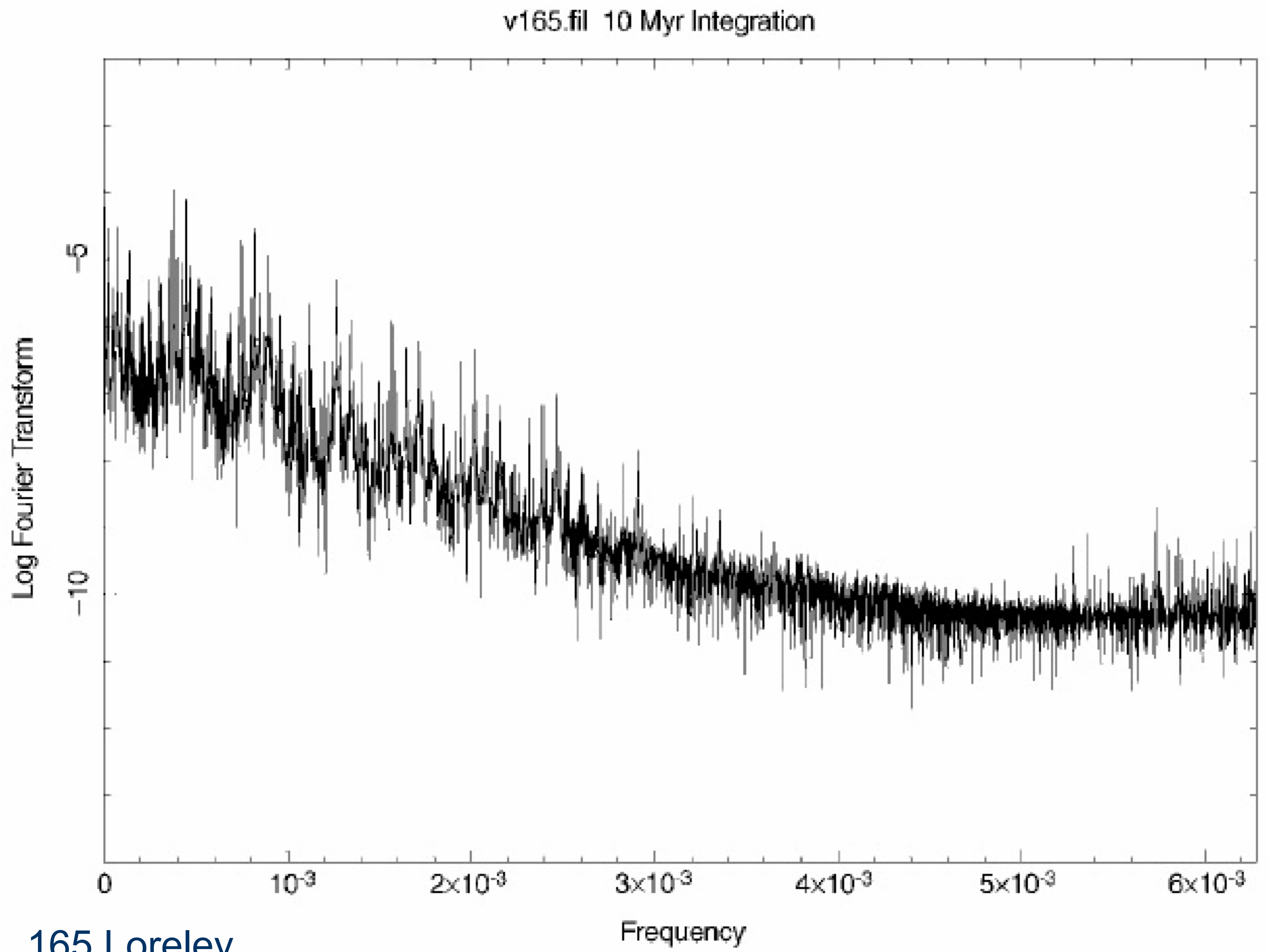
1223 Neckar

Haotični eksponencijalno stabilni



305 Gordonia, režim Nehoroševa

Haotično difuzivni (Chirikov)



165 Loreley

Budući rad

- Bolja aproksimacija Hamiltonijana
- Primeniti teoremu Nehoroševa u izvornom obliku

HVALA!