

MCMC Parameters Estimation for Inflationary Cosmological Models

Milan Milošević¹

in collaboration with
Goran Djordjević¹, Đorđe Milanović²,
Marko Stojanović³, and Dragoljub D. Dimitrijević¹

¹Department of Physics, Faculty of Sciences and Mathematics, University of Niš

²Institute of Physics Belgrade, University of Belgrade, Serbia

³Faculty of Medicine, University of Niš, Serbia

Serbian Astronomical Conference, 2026

Outline of the talk

1 Inflation and what we measure

slow roll, the observables n_s and r , targets from Planck 2018, ACT DR6 and DESI

2 From a model to (n_s, r)

three inflationary cosmologies, one numerical pipeline: `inflationSim`

3 Bayesian parameter estimation with MCMC

posterior, the Metropolis–Hastings algorithm, convergence diagnostics

4 Results

posteriors in the (n_s, r) plane, correlations between parameters, new data

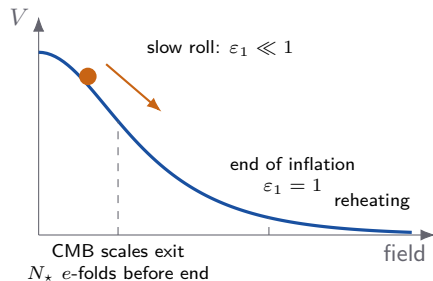
5 How much does the (n_s, r) method matter?

eight ways to compute (n_s, r) from the same background

6 A complementary machine-learning tool, and conclusions

Cosmic inflation in one slide

- A short epoch of **accelerated expansion** in the very early Universe, lasting $\gtrsim 60$ e -folds.
- Explains why the Universe is so **flat** and **uniform** on large scales.
- Driven by a field that **slowly rolls** down its potential V .
- Quantum fluctuations are stretched beyond the horizon and become the **seeds of CMB anisotropies** and of all structure.
- The same process produces **primordial gravitational waves**.



The potentials in this talk belong to the family $V \propto 1/\cosh$: a plateau followed by a decay.

What we measure: n_s and r

Primordial power spectra near a pivot scale k_* :

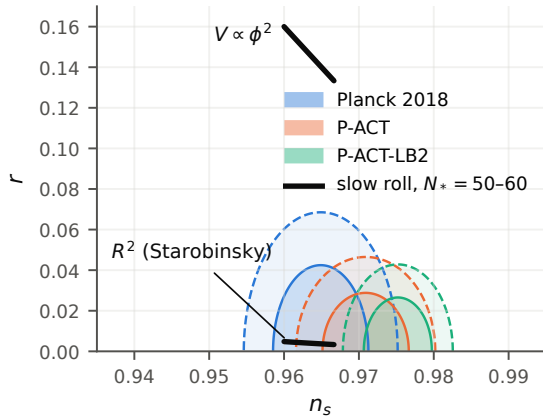
$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_*} \right)^{n_s-1}, \quad r = \frac{\mathcal{P}_t(k_*)}{\mathcal{P}_s(k_*)}$$

- n_s – **scalar spectral index**; $n_s = 1$ is scale invariant, data give $n_s < 1$.
- r – **tensor-to-scalar ratio**; so far only upper bounds.

Data set	n_s	r (95%)
Planck 2018	0.9649 ± 0.0042	< 0.056
P-ACT	0.9709 ± 0.0038	< 0.038
P-ACT-LB2	0.9752 ± 0.0030	< 0.035

P-ACT: Planck + ACT DR6; LB2: + CMB lensing, DESI DR2 BAO.

Contours: Gaussian summaries of the constraints, used as targets in this work. Planck Collaboration (2020); ACT DR6: arXiv:2503.14452, 2503.14454.



Each model maps its parameters to points in this plane.

From a model to (n_s, r) : slow-roll principle

Hubble-flow parameters measure how slowly the expansion changes:

$$\varepsilon_1 = -\frac{\dot{H}}{H^2}, \quad \varepsilon_{i+1} = \frac{\dot{\varepsilon}_i}{H\varepsilon_i}$$

Inflation: $\varepsilon_1 < 1$; slow roll: $|\varepsilon_i| \ll 1$.

All quantities are evaluated at horizon exit, $N_\star$ e -folds before the end of inflation.

Leading order (canonical field):

$$n_s \simeq 1 - 2\varepsilon_1 - \varepsilon_2, \quad r \simeq 16\varepsilon_1$$

Non-canonical fields (tachyon, DBI):
perturbations propagate with a **sound speed**
 $c_s \leq 1$.

$$\mathcal{P}_s \simeq \frac{H^2}{8\pi^2\varepsilon_1 c_s}, \quad \mathcal{P}_t \simeq \frac{2H^2}{\pi^2}$$
$$r \simeq 16\varepsilon_1 c_s$$

The chain of computation

model $\rightarrow$ background $H(t) \rightarrow \varepsilon_i, c_s$ at $N_\star \rightarrow (n_s, r)$

Three models, one family of potentials

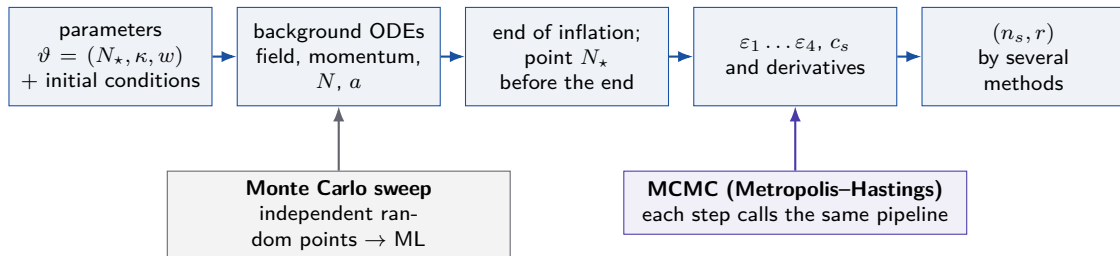
Model	Field and energy density	Expansion rate	Free parameters
Tachyon tachyon field, standard cosmology	$V = \text{sech}(w\theta)$ $\rho = \sqrt{V^2 + \pi_\theta^2}$	$H^2 = \frac{\kappa^2 \rho}{3}$	$N_\star, \kappa, w$
Braneworld DBI-type field on a Randall–Sundrum II brane	$\chi = \cosh^{1/4} \varphi$ $\rho = \chi^{-4} \sqrt{1 + \chi^8 \pi_\varphi^2}$	$H^2 = \frac{\kappa^2 \rho}{3} \left(\chi' + \frac{\kappa^2 \rho}{12} \right)$	$N_\star, \kappa$
Holographic tachyon field, holographic cosmology	$V = \text{sech}(w\theta)$ $\rho = V \sqrt{1 + \eta^2}$	$H^2 = 2 - 2\sqrt{1 - \frac{\kappa^2 \rho}{3}}$	$N_\star, \kappa, w$

κ : coupling (gravitational/brane scale); w : width of the potential;
 $N_\star \in [60, 120]$: e -folds between horizon exit and the end of inflation.

Dimensionless units. Bilić et al., IJMPA 32 (2017) 1750039; JCAP 08 (2019) 034; Bertini et al., arXiv:2007.02332.

Same type of matter, **different cosmology**:
 the Friedmann equation is what changes.

One numerical pipeline for every model



inflationSim: C++20, Boost.odeint
(Runge–Kutta–Fehlberg 7(8)), OpenMP.
A new model only supplies its equations, H and V .

One evaluation = one full integration (~ 0.1 – 1 s).
Failed trajectories are simply rejected.

Parameter estimation: from data to a posterior

Bayes' theorem

$$p(\vartheta \mid D) \propto \mathcal{L}(D \mid \vartheta) \pi(\vartheta)$$

- $\vartheta = (N_\star, \kappa, w)$: model parameters.
- $\pi(\vartheta)$: **prior**, uniform inside a box.
- $\mathcal{L}$: **likelihood**, how well $n_s(\vartheta)$, $r(\vartheta)$ match the data.
- $p(\vartheta \mid D)$: **posterior**; gives credible intervals and correlations.

Likelihood in the (n_s, r) plane

$$\ln \mathcal{L} = -\frac{1}{2} \left[\frac{(n_s - n_s^{\text{obs}})^2}{\sigma_{n_s}^2} + \frac{r^2}{\sigma_r^2} \right], \quad \sigma_r = \frac{r_{95}}{2}$$

Why sampling?

$p(\vartheta \mid D)$ has no closed form: every value of $\mathcal{L}$ needs a full numerical background. A grid in 3 parameters is wasteful; the posterior may be a narrow, curved ridge. **MCMC draws samples directly from $p(\vartheta \mid D)$.**

Markov chain Monte Carlo: the Metropolis–Hastings algorithm

- 1 Propose a step: $\vartheta' = \vartheta + \delta$, $\delta \sim \mathcal{N}(0, \text{diag } \sigma^2)$.
- 2 Accept with probability
 $A = \min(1, p(\vartheta' | D)/p(\vartheta | D))$.
- 3 If rejected, **repeat** the current state.

Better points are always accepted, worse ones sometimes.

After a **burn-in**, the visited states are distributed as the posterior.

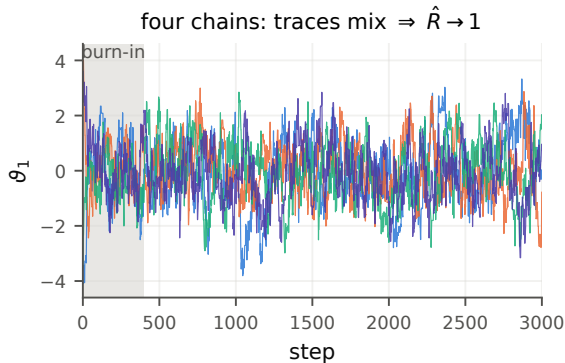
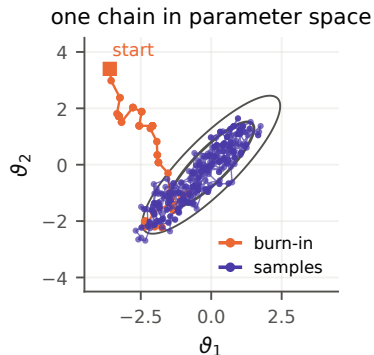


Illustration on a correlated 2D Gaussian: the chain forgets its starting point and then explores the target region.

Running the chains: burn-in, adaptation, convergence

Setup (per model and data set)

- 4 independent chains, random starts
- 2×10^4 steps (10^4 for **Braneworld**), first 15% burn-in
- step size adapted during burn-in to acceptance ≈ 0.23

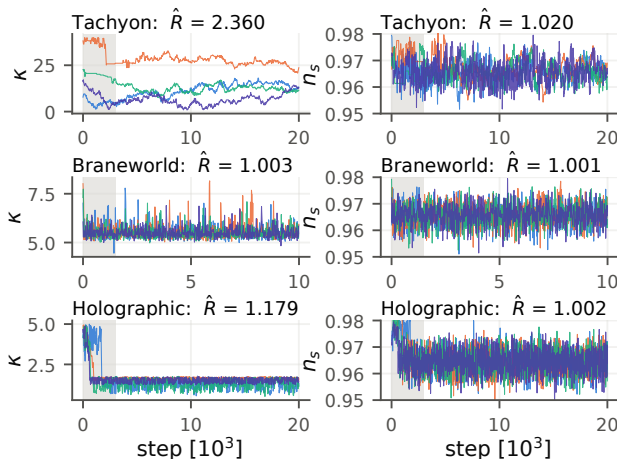
Gelman–Rubin $\hat{R}$: spread between chains vs. within chains; $\hat{R} \rightarrow 1$ when they agree.

Reading the traces

Braneworld: chains indistinguishable.

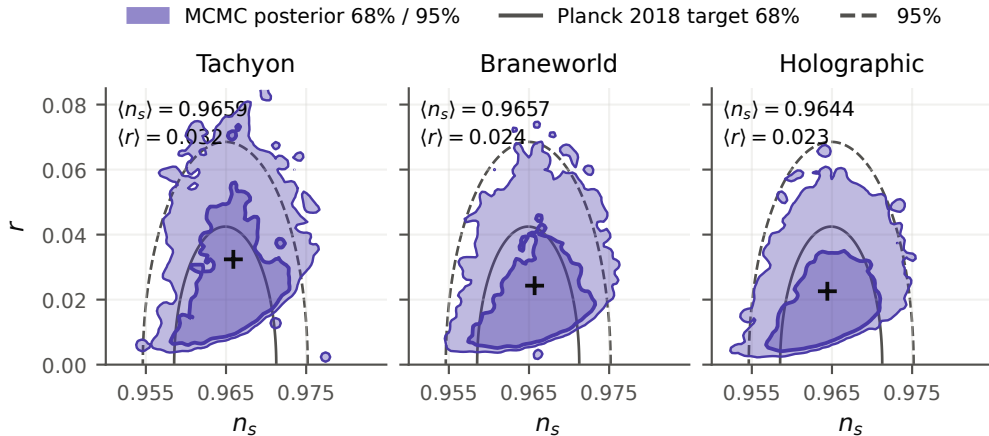
Holographic: n_s agrees; κ , w mix more slowly.

Tachyon: chains sit on different parts of a long ridge.



Planck 2018 target; four chains in color; gray: burn-in.

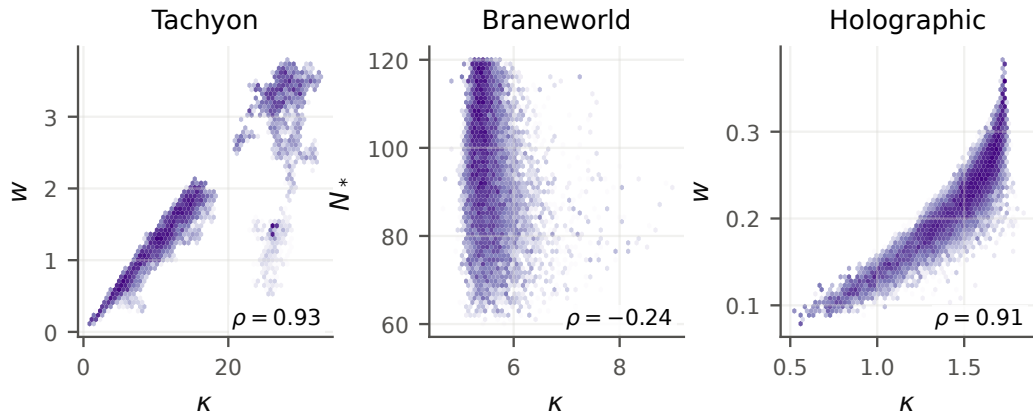
Result 1: posteriors in the (n_s, r) plane, Planck 2018



- All three posteriors are centred in the Planck 68% contour: $\langle n_s \rangle \approx 0.965$.
- Posterior mean $\langle r \rangle = 0.023$ – 0.032 , with a broad spread (SD 0.014–0.020); below the bound.

Pooled post-burn-in states of 4 chains, n_s and r from the PS approximation; +: posterior mean.

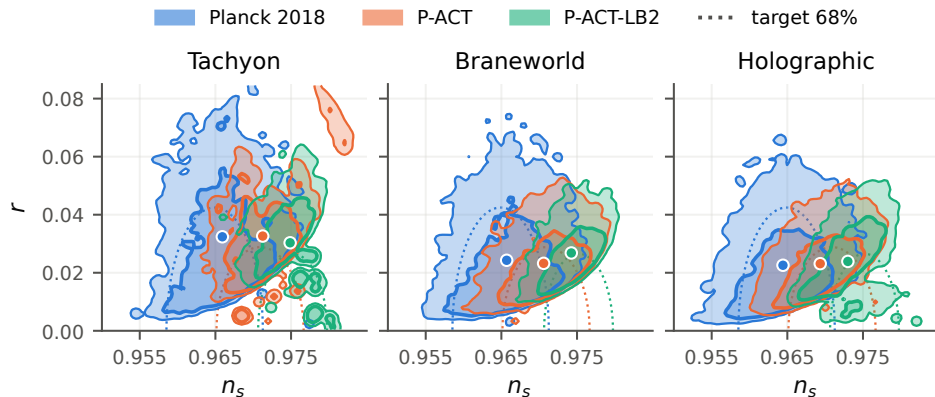
Result 2: what MCMC adds – correlations between parameters



- **Tachyon** and **Holographic**: the data fix a **combination** of κ and $w \Rightarrow$ a narrow ridge ($w \approx 0.12 \kappa$ and $w \approx 0.15 \kappa$, $\rho > 0.9$).
- **Braneworld**: $\kappa \approx 5.5$ is sharply determined, while N_* spans the whole prior range. A box of parameter ranges cannot describe such shapes; the posterior does.

Planck 2018 target; density of posterior samples (log scale); ρ : Pearson correlation.

Result 3: from Planck to ACT DR6 and DESI



$\langle n_s \rangle / \langle r \rangle$	Planck 2018	P-ACT	P-ACT-LB2
Tachyon	0.9659 / 0.032	0.9713 / 0.033	0.9749 / 0.030
Braneworld	0.9657 / 0.024	0.9706 / 0.023	0.9742 / 0.027
Holographic	0.9644 / 0.023	0.9693 / 0.023	0.9730 / 0.024

All three models move toward higher n_s without a conflict in r .

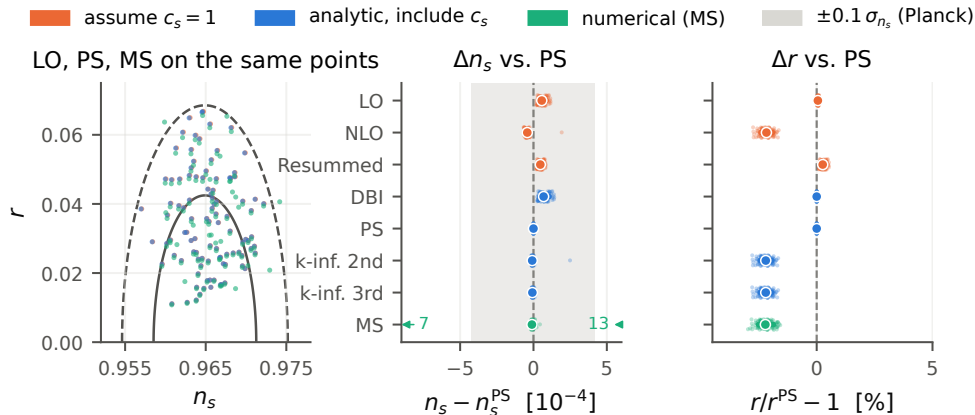
Eight ways to compute (n_s, r) from the same background

Method	Idea	uses c_s
LO	leading-order slow roll: $n_s = 1 - 2\varepsilon_1 - \varepsilon_2$, $r = 16\varepsilon_1$	—
NLO	second-order slow-roll corrections, model-specific coefficients	—
Resummed	sums all powers of ε_1 in the leading terms	—
DBI	leading order with sound-speed flow, $r = 16\varepsilon_1 c_s$	✓
PS	leading amplitudes $\mathcal{P}_s$, $\mathcal{P}_t$; n_s from their scale dependence, used in MCMC	✓
k-inf. 2nd / 3rd	high-order slow-variation expansion of k -inflation (Zhu et al. 2014)	✓
MS	numerical evolution of the Mukhanov–Sasaki mode equation	✓

$$\text{MS: } v_q'' + \left(c_s^2 q^2 - \frac{z''}{z} \right) v_q = 0, \quad z = \frac{a\sqrt{2\varepsilon_1}}{c_s}, \quad \mathcal{P}_s = \frac{q^3 |v_q|^2}{2\pi^2 z^2}$$

PS is algebraic and fast, so it is the natural choice inside a chain of 10^4 – 10^5 evaluations. Formulas: backup B2.

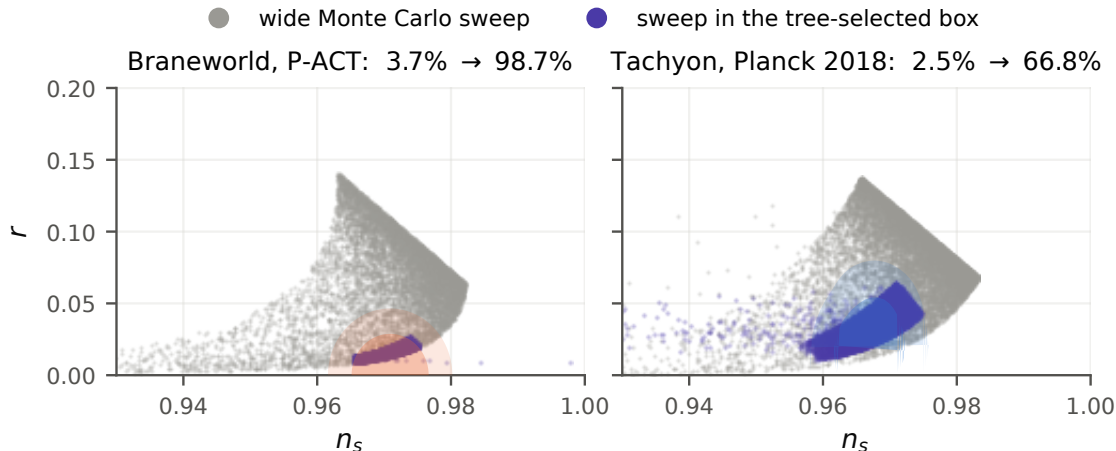
Result 4: how much do the methods differ?



- All methods: median $|\Delta n_s| \lesssim 0.7 \times 10^{-4}$ ($< 0.02 \sigma_{n_s}$).
- r : LO, Resummed and DBI agree with PS; NLO, k -inflation and MS are lower by $\approx 2.2\%$, in mutual agreement.
- MS: individual points scatter more (20/100 with $|\Delta n_s| > 5 \times 10^{-4}$).

100 points of the **Holographic** model; PS used as the common reference. Numbers: backup B3.

A complementary tool: machine-learning range finder



A decision tree trained on a wide Monte Carlo sweep proposes a box of parameter ranges; a new sweep inside that box lands mostly in the target region. Fast and readable, however, it is still a box, not a posterior.

Fraction of points inside the 68% contour, before → after narrowing.

Conclusions

- ① `inflationSim` computes (n_s, r) for different inflationary cosmologies with one pipeline; Metropolis–Hastings MCMC now runs on top of it.
- ② MCMC gives full posteriors: **Tachyon**, **Braneworld** and **Holographic** all reach the Planck 2018 and ACT DR6 regions, with $\langle r \rangle \approx 0.02\text{--}0.03$; $\langle n_s \rangle$ follows the data upward.
- ③ The posteriors reveal **degeneracies** (κ – w ridges); convergence diagnostics are an essential part of the result.
- ④ On the same background, the (n_s, r) methods differ by $\lesssim 10^{-4}$ in n_s ($< 0.02\sigma$, median) and by $\approx 2\%$ in r : below today's uncertainties.
- ⑤ Next: more parameters and other applications (primordial black holes, dark matter).

Supported by ICTP–SEENET-MTP project NT-03 “Cosmology - classical and quantum challenges” and by the Ministry of Science, Technological Development and Innovation of the Republic of Serbia (451-03-34/2026-03/200124).

Thank you for your attention.

B1 Equations of motion: the **Tachyon** model

Tachyon field θ with canonical momentum π_θ , potential $V = \text{sech}(w\theta)$:

$$\rho = \sqrt{V^2 + \pi_\theta^2}, \quad p = -\frac{V^2}{\rho}$$

Hamilton's equations, state vector $(\theta, \pi_\theta, N, a)$:

$$\dot{\theta} = \frac{\pi_\theta}{\rho}, \quad \dot{\pi}_\theta = -3H\pi_\theta - \frac{VV'}{\rho}, \quad \dot{N} = H, \quad \dot{a} = Ha$$

Friedmann equation: $H^2 = \kappa^2 \rho / 3$.

Same structure for every model

Each model class supplies its own equations, H and V ; the common base class integrates the background, finds the end of inflation and the point $N_\star$, and computes $\varepsilon_{1\dots 4}$, c_s and (n_s, r) .

B2 Analytical formulas

Method	Expression
LO	$n_s = 1 - 2\varepsilon_1 - \varepsilon_2, \quad r = 16\varepsilon_1$
NLO	$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - [2\varepsilon_1^2 + (2C + 3 - 2\alpha)\varepsilon_1\varepsilon_2 + C\varepsilon_2\varepsilon_3], \quad r = 16\varepsilon_1(1 + C\varepsilon_2 - 2\alpha\varepsilon_1),$ $C = \gamma_E + \ln 2 - 2$ (model-specific coefficients)
Resummed	$S = \sum_{j \geq 1} \varepsilon_1^j, \quad n_s = 1 - 2S - \varepsilon_2, \quad r = 16S$
DBI	$n_s = 1 - 2\varepsilon_1 - \varepsilon_2 - s, \quad r = 16\varepsilon_1 c_s, \quad s = \dot{c}_s / (H c_s)$
PS	$\mathcal{P}_s = \frac{H^2}{8\pi^2 \varepsilon_1 c_s}, \quad \mathcal{P}_t = \frac{2H^2}{\pi^2}, \quad n_s - 1 = \frac{-2\varepsilon_1 - \varepsilon_2 + q_1}{1 - \varepsilon_1}, \quad r = 16\varepsilon_1 c_s, \quad q_1 = -s$
k-inf. 2nd / 3rd	expansions in ε_i and sound-flow parameters q_i to second / third order; Zhu, Wang, Cleaver, Kirsten & Sheng, PRD 90, 103517 (2014), Eqs. (4.29), (4.34)
MS	numerical modes on the stored background; n_s from $d \ln \mathcal{P}_s / d \ln q$ between neighbouring modes

B3 Method comparison: numbers

Method	$\text{med}(n_s)$	$\text{med}(r)$	$\text{med}(\Delta n_s)$ [10^{-4}]	$\text{SD}(\Delta n_s)$ [10^{-4}]	$\text{med}(\Delta r/r_{\text{PS}})$	$\text{corr}(n_s, n_s^{\text{PS}})$
LO	0.96540	0.03337	+0.56	0.25	+0.05%	0.99997
NLO	0.96531	0.03267	-0.42	0.26	-2.17%	0.99997
Resummed	0.96539	0.03344	+0.47	0.17	+0.26%	0.99999
DBI	0.96541	0.03336	+0.69	0.30	+0.00%	0.99996
PS	0.96535	0.03336	+0.00	0.00	+0.00%	1.00000
k-inf. 2nd	0.96534	0.03266	-0.09	0.26	-2.20%	0.99997
k-inf. 3rd	0.96534	0.03266	-0.08	0.05	-2.20%	1.00000
MS	0.96551	0.03265	-0.11	13.62	-2.22%	0.91074
Custom	0.96534	0.03266	-0.08	0.05	-2.20%	1.00000

100 points of the **Holographic** model, $V = \text{sech}(w\theta)$; differences are paired point by point, PS is the reference.
Custom: model-specific spline evaluation, not shown in the main figure.

B4 Convergence diagnostics for all runs

Model / data set	$\hat{R}(N_\star)$	$\hat{R}(\kappa)$	$\hat{R}(w)$	$\hat{R}(n_s)$	$\hat{R}(r)$
Tachyon / Planck 2018	1.211	2.360	2.033	1.020	1.162
Tachyon / P-ACT	1.353	2.702	2.449	1.160	1.111
Tachyon / P-ACT-LB2	1.611	2.706	2.460	1.249	1.442
Braneworld / Planck 2018	1.004	1.003	—	1.001	1.005
Braneworld / P-ACT	1.003	1.001	—	1.001	1.002
Braneworld / P-ACT-LB2	1.001	1.001	—	1.001	1.001
Holographic / Planck 2018	1.012	1.179	1.176	1.002	1.009
Holographic / P-ACT	1.012	1.087	1.087	1.001	1.007
Holographic / P-ACT-LB2	1.082	1.219	1.137	1.034	1.452

Rank-normalized split/folded $\hat{R}$ (Vehtari et al. 2021). **Tachyon** and **Holographic**: $4 \times 17\,000$ post-burn-in states per run; **Braneworld**: $4 \times 8\,500$. **Braneworld** has w fixed.

B5 Posterior means and standard deviations

Model / data set	n_s	r
Tachyon / Planck 2018	0.9659 ± 0.0040	0.0324 ± 0.0196
Tachyon / P-ACT	0.9713 ± 0.0038	0.0326 ± 0.0146
Tachyon / P-ACT-LB2	0.9749 ± 0.0027	0.0303 ± 0.0131
Braneworld / Planck 2018	0.9657 ± 0.0041	0.0243 ± 0.0154
Braneworld / P-ACT	0.9706 ± 0.0035	0.0232 ± 0.0100
Braneworld / P-ACT-LB2	0.9742 ± 0.0026	0.0268 ± 0.0088
Holographic / Planck 2018	0.9644 ± 0.0041	0.0226 ± 0.0136
Holographic / P-ACT	0.9693 ± 0.0035	0.0231 ± 0.0097
Holographic / P-ACT-LB2	0.9730 ± 0.0030	0.0239 ± 0.0104

All repeated states retained; n_s and r from the PS approximation.

B6 MCMC settings

	Tachyon	Braneworld	Holographic
prior $N_\star$	[60, 120]	[60, 120]	[60, 120]
prior κ	[0, 40]	[0, 10]	[0, 5]
prior w	[0, 5]	fixed 1	[0, 1]
initial steps ($N_\star, \kappa, w$)	1.5, 0.26, 0.031	11.2, 0.56	4.1, 0.20, 0.041
steps / burn-in	$2 \cdot 10^4 / 3 \cdot 10^3$	$10^4 / 1.5 \cdot 10^3$	$2 \cdot 10^4 / 3 \cdot 10^3$
chains	4	4	4

Adaptation every 100 steps during burn-in, $\sigma \rightarrow \sigma \exp[0.05(\text{acc} - 0.234)]$; fixed afterwards. Likelihood targets: Planck 2018, P-ACT, P-ACT-LB2 (slide 4), $\sigma_r = r_{95}/2$. Initial steps above: Planck 2018 runs; P-ACT and P-ACT-LB2 runs start from similar values (1.0–1.1, 0.17–0.18, 0.021 / 11, 0.54–0.56 / 3.2–3.5, 0.16–0.17, 0.032–0.035).

B7 References

- Bilić et al., *Tachyon inflation in an AdS braneworld with backreaction*, Int. J. Mod. Phys. A 32 (2017) 1750039.
- Bilić et al., *Tachyon inflation in the holographic braneworld*, JCAP 08 (2019) 034.
- Bertini, Bilić & Rodrigues, *Primordial perturbations and inflation in holographic cosmology*, arXiv:2007.02332.
- Zhu, Wang, Cleaver, Kirsten & Sheng, *Power spectra and spectral indices of k -inflation: high-order corrections*, Phys. Rev. D 90, 103517 (2014).
- Planck Collaboration, *Planck 2018 results. X. Constraints on inflation*, A&A 641 (2020) A10.
- ACT DR6: arXiv:2503.14452 (power spectra and likelihoods), arXiv:2503.14454 (extended models).
- Vehtari, Gelman, Simpson, Carpenter & Bürkner, *Rank-normalization, folding, and localization: an improved $\hat{R}$* , Bayesian Analysis 16 (2021) 667.

Data: campaign of 4 October 2026 (36 chains, 5.1×10^5 post-burn-in states; 100-point method sample of the **Holographic** model); narrowed ML sweeps from 25 September 2026. Figures: SAC/scripts/fig_v10.py.