

A systematic survey of the von Zeipel-Lidov-Kozai resonance across the Solar System: from inside Jupiter's orbit to the trans-Neptunian region

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Astronomical Observatory Belgrade, Serbia

5 - 8 October 2026

XXI Serbian Astronomical Conference

Palace of science, Kralja Milana 11, Belgrade, Serbia

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Abstract

The von Zeipel–Lidov–Kozai (ZLK) mechanism drives secular oscillations of eccentricity and inclination that shape the orbital architecture of small bodies throughout the Solar System, but a catalog-wide observational census of ZLK-resonant bodies has so far been lacking. We present two complementary systematic surveys that together cover essentially the entire numbered small-body population: 1037 numbered trans-Neptunian objects (TNOs) with semi-major axes $a > 30$ AU and 863471 asteroids with $a < 4.9$ AU from the AstDyS catalog. Orbits were integrated in a full N-body model (Sun, eight planets, Pluto) using the *resonances* python package, with integration spans ranging from 100 kyr up to 300 Myr for some TNO candidates. Concurrent membership in two-body and three-body mean-motion resonances (MMRs) was tested for every ZLKR candidate.

In the trans-Neptunian region, 81 objects ($\approx 7.8\%$) display clear ZLK libration and 23 additional objects exhibit transient behavior. Every confirmed liblator is simultaneously trapped in a two-body MMR with Neptune – direct empirical confirmation of long-standing theoretical predictions. The most populated resonances are 2N-3 (58 objects), 4N-7 (15), 1N-2 (9), and 3N-5 (7); libration centers cluster near 90° and 270° for all populated MMRs except 1N-2, where they are shifted to approximately 120° , 150° , 300° , and 330° .

Inside Jupiter's orbit, in contrast, we identify 201 ($< 0.03\%$) confirmed ZLKR asteroids (163 with stable libration parameters, 38 irregular) and an additional 26309 candidates. The population is overwhelmingly planet-crossing: 144 near-Earth asteroids and 28 Mars-crossers, while the stable main belt is almost empty of confirmed librators. Only 11 of the 201 are simultaneously in an MMR, and roughly $1/3$ have inclinations below the classical quadrupole critical value of $\approx 39.2^\circ$.

Taken together, these results show that the tight MMR–ZLK coupling found beyond Neptune is not a universal feature of the Solar System. Inside Jupiter's orbit, direct planetary perturbations, MMR overlap, and competition with strong secular resonances appear to decouple (and possibly even anti-correlate) ZLK libration from MMR capture, making the MMR–ZLK relationship a regime-dependent rather than universal property.

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- ▶ The von Zeipel-Lidov-Kozai (ZLK) mechanism
- ▶ von Zeipel (1910), Lidov (1961), Kozai (1962)
- ▶ ZLK resonance is closely tied to mean motion resonances (MMRs).
- ▶ Gallardo 2006 – the locking of the semi-major axis inside an MMR allows ZLK libration.
- ▶ Lei et al. (2022) showed that the libration centers can then shift away from 90° and 270° .
- ▶ Inside Jupiter's orbit (Michel and Thomas 1996, Gronchi and Milani 1999).
- ▶ Carruba et al. (2026) identified ZLK-resonant co-orbitals of the terrestrial planets.
- ▶ Morbidelli (2002), Shevchenko (2017), Sidorenko (2018) $\Rightarrow$ we treat the ZLK effect as a nonlinear resonance.
- ▶ A catalog-wide census of the ZLK resonance had not been carried out in either region.
- ▶ Throughout this talk $\sigma \equiv \omega$.

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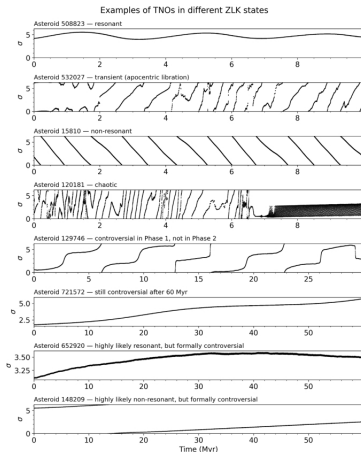


Figure: Resonant arguments for TNOs with different resonant statuses (from top to bottom): resonant; transient - apocentric libration; non-resonant; chaotic; controversial in Phase 1, not in Phase 2; still controversial after 60 Myr, but non-resonant after 120 Myr; highly likely resonant after Phase 3, but formally classified as resonant after Phase 4; highly likely non-resonant after Phase 3 and formally classified as non-resonant after Phase 5.

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Dynamically interesting objects:

- (652920) 2014 GR₅₃, a distant TNO, $a \approx 202.8$ au - in ZLKR and in 1N-18.
- (523700) 2014 GM₅₄, low c_1 - 0.54 against 0.92 on average.
- (585899) 2020 HM₉₈, retrograde, $i \approx 138^\circ$, with a 22 Myr ZLK cycle; trapped in 2N+9, 5N-24, ...
- (523695) 2014 GS₅₃, multiple captures in MMRs, and transient in ZLKR.

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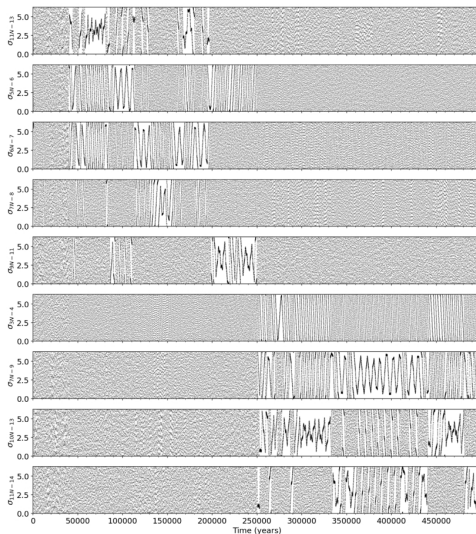


Figure: Multiple captures in MMRs of the TNO 523695 (one object, several resonant angles - including high-order and high-integer MMRs - with capture, release and re-capture within a single integration).

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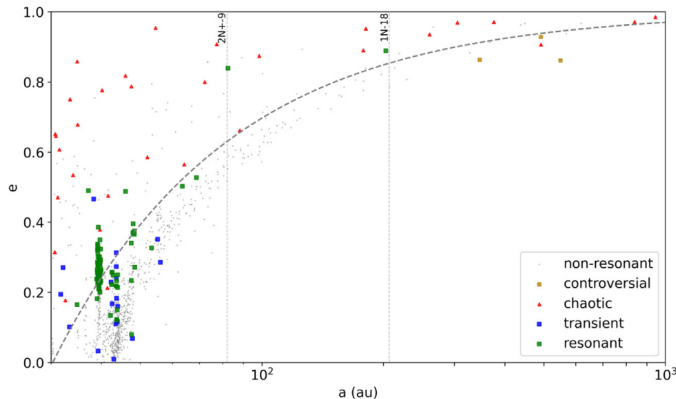


Figure: TNO sample in the a - e plane.

- 81 of **1037** numbered TNOs are ZLKR $\approx 7.8\%$, 100% of 81 are in MMRs; 23 transient objects exhibiting alternating libration and circulation; 896 (86.4%) are non-resonant; 4 remain controversial even after 300 Myr of integration, and 33 have chaotic behavior preventing from the classification.

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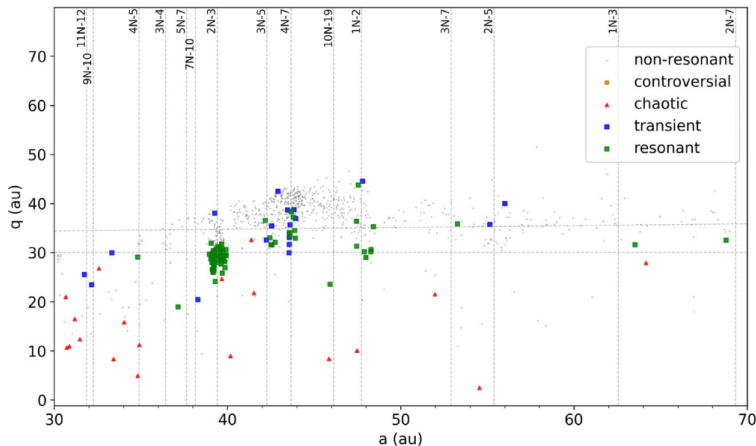


Figure: Plane of semi-major axis versus perihelion distance. The dashed horizontal lines indicate the perihelion distances of the chaotic border and Neptune, respectively (from top to bottom)

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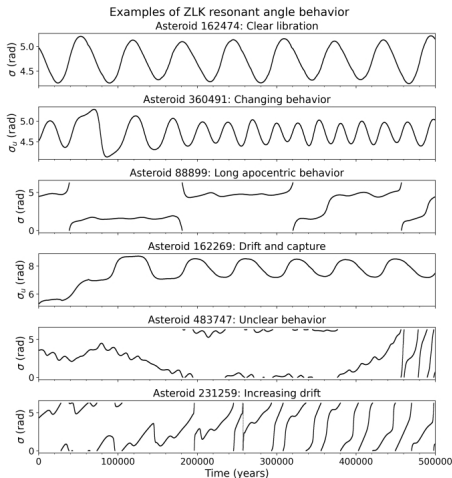


Figure: Libration is rarely textbook-clean. Clear libration, changing behaviour, long apocentric libration, drift and capture, unclear behaviour, increasing drift - all over 500 kyr.

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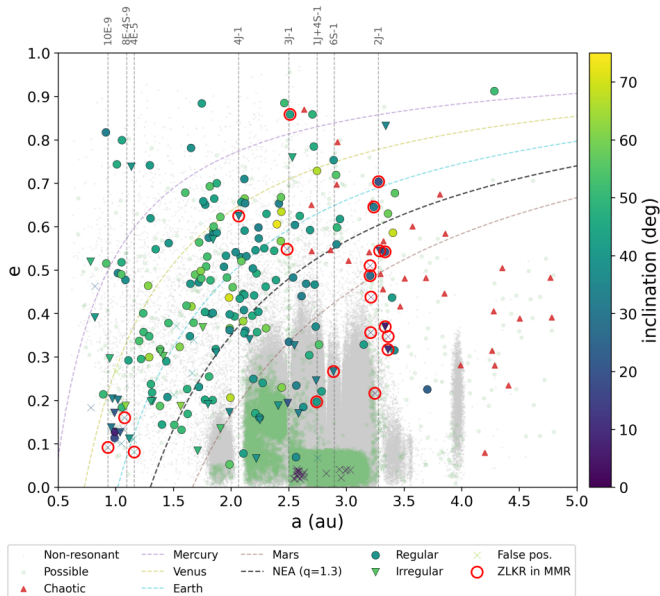
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Figure: ZLKR librators inside 5 au, colour coded by inclination.

Results for asteroids inside Jupiter's orbit

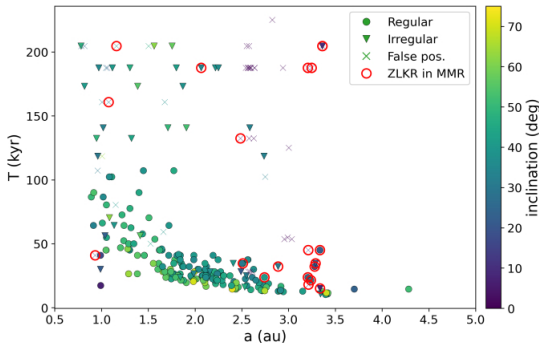


Figure: Libration period as a function of the semimajor axis for confirmed ZLKR asteroids, colored by orbital inclination.

- 201 of 863471 asteroids are in ZLKR ($< 0.03\%$); 144 of 201 are NEA, 28 are MC, **only** 11 ($< 6\%$) are in MMRs;
- 26309 of 863471 possibly/temporarily in ZLKR.
- Inside Jupiter's orbit, unlike in the TNO region, there are false positives: formally satisfies the libration criterion, but shows a strong, monotonic drift of the libration center, no $e - i$ anti-correlation or a very slow evolution of σ that is best interpreted as near-resonant.

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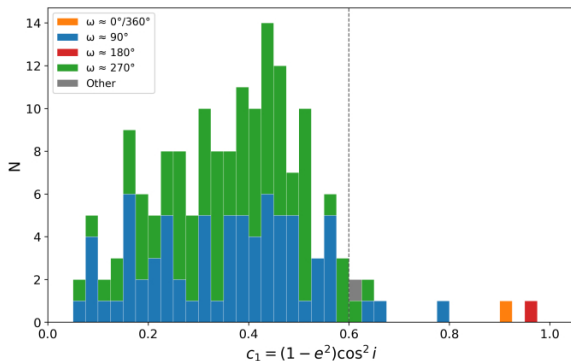


Figure: Stacked histogram of the Kozai integral $c_1 = (1 - e^2)\cos^2 i$ for the 163 regular ZLKR asteroids, color-coded by libration center. The vertical dashed line at $c_1 = 3/5 = 0.6$ is the classical quadrupole threshold above which libration of ω is forbidden in the restricted test-particle problem. Bars to the right of the threshold demonstrate the empirical extension of ZLK libration to $c_1 > 3/5$, mainly via nonclassical libration centers.

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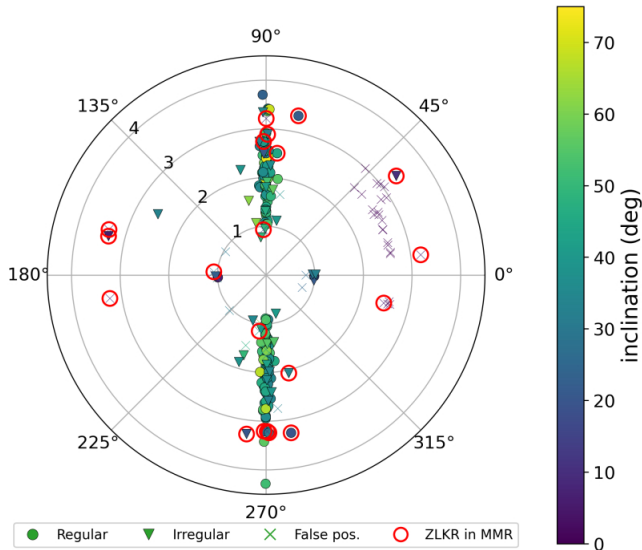


Figure: Polar representation of the libration centers, ω_c , of confirmed ZLKR asteroids. The angular coordinate is ω_c , the radial coordinate is the semimajor axis (expressed in au), and the color encodes the orbital inclination.

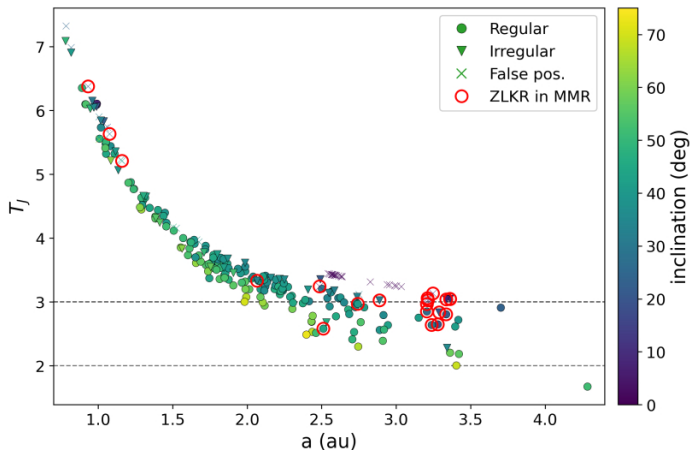


Figure: Tisserand parameter with respect to Jupiter, T_J , as a function of semimajor axis for confirmed ZLKR asteroids. Colors encode the orbital inclination. The horizontal dashed line at $T_J = 3$ is the canonical asteroid-comet boundary.

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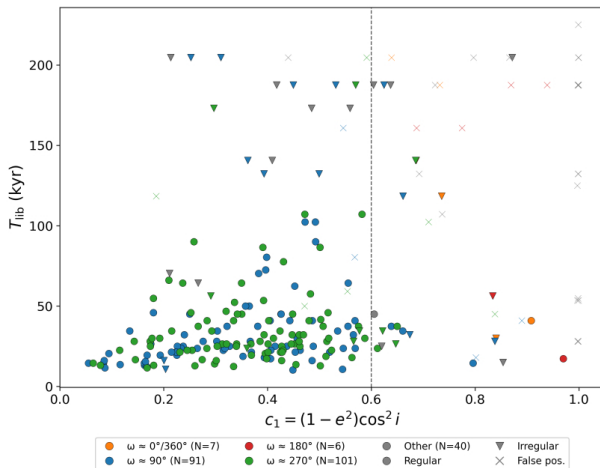


Figure: Libration period, T_{lib} , versus the Kozai integral c_1 , colored by libration-center category.

Conclusions

- ▶ Beyond Neptune the ZLK resonance and the MMR network are tightly coupled: every ZLK-resonant TNO is locked in a two-body MMR with Neptune, and the locking of the semi-major axis creates the conditions for the ZLK libration.
- ▶ For 2N-3, 4N-7, 3N-5 and 4N-5 the libration centres are near 90° and 270° , as expected; for 1N-2 they shift to approximately 120° , 150° , 300° and 330° . 1N-x MMRs are indeed strong: there is a TNO, (652920) 2014 GR₅₃, trapped in 1N-18.
- ▶ Inside Jupiter's orbit this coupling is almost absent. From the 863471 asteroids inside Jupiter orbit, 201 are confirmed ZLK-resonant (163 regular and 38 irregular), less than 0.03% of the sample.
- ▶ Fewer than 6% of the ZLK-resonant asteroids are in an MMR, the surviving pairs concentrate in the 2J-1 resonance, and the ZLK resonance is instead a planet-crossing phenomenon (144 are near-Earth asteroids and 28 are Mars-crossers, while the main belt is almost empty) that reaches below the classical critical inclination.
- ▶ The interplay and overlapping of different resonances, chaos, to eccentricity pumping onto planet-crossing orbits, and to competition with strong secular resonances in the main belt appears to suppress long-lived ZLK libration.

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This work has been supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia via the contract 451-03-33/2026-03/200002.

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