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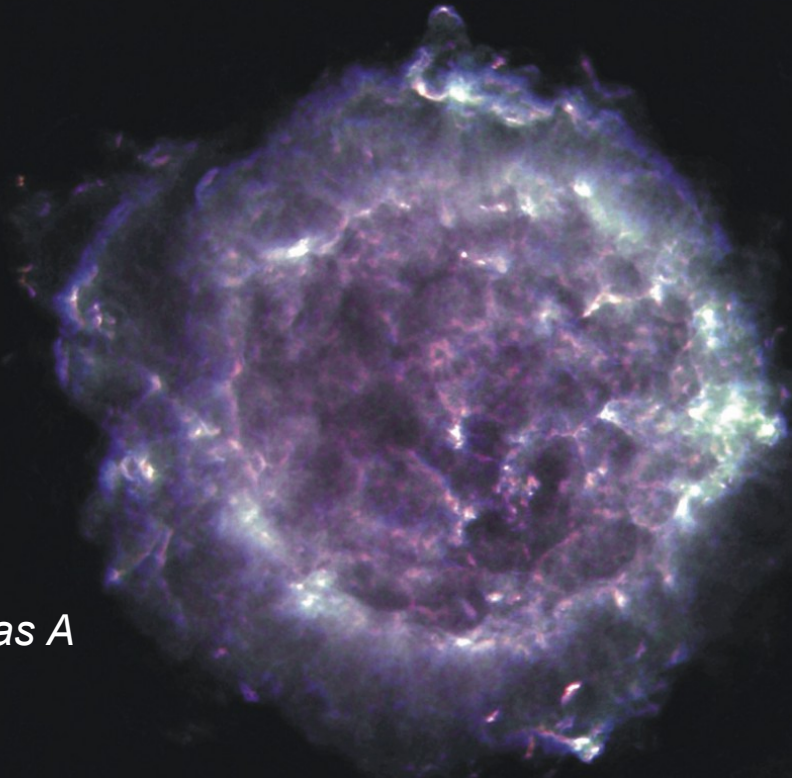
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RADIO EVOLUTION OF SUPERNOVA REMNANTS



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RADIO SUPERNOVA REMNANTS

- numerous observations of supernova remnants in radio (VLA, ATCA, ALMA, Planck), but also at other wavelengths
 - Galactic (~ 300 , [Green 2025](#)) and extragalactic radio SNRs catalogues ([Urošević et al. 2005](#), [Bozzetto et al. 2017](#))
 - ultra-relativistic particles (electrons) + amplified/compressed magnetic field $\rightarrow$ radio synchrotron emission
 - *non-thermal* !
- Cas A
- 
- Diffusive shock acceleration (DSA) - macroscopic approach: [Axford et al. \(1977\)](#), [Krymsky \(1977\)](#) and [Blandford & Ostriker \(1978\)](#), microscopic approach: [Bell \(1978\)](#)
 - compression $r=4$, energy spectrum with power-law slope $\Gamma=2$, radio spectrum with spectral index $\alpha = (\Gamma-1)/2 = 0.5$.

A LITTLE BIT OF HISTORY ...

- discovered by Victor Hess in a balloon experiment in 1912.
- ultra-relativistic particles: protons, α -particles, heavier nuclei, electrons...



*Victor Hess
(1883 – 1964)*

- $p^+ : e^- = 100 : 1$
- won a Nobel prize in Physics in 1936 “for his discovery of cosmic radiation” (together with C.D. Anderson for his discovery of the positron)
- primary and secondary
- all particle spectrum - a power-law with energy index $\Gamma \approx 2.7$ up to $\sim 10^{15}$ eV, the so-called “knee”
- source: supernova remnants, $\Gamma \approx 2$



- $\Sigma - D$ relation

$$\Sigma_\nu = AD^{-\beta}$$

- **empirical**, surface brightness = flux density / solid angle
- distance determination, $D = \theta d$ (Shklovsky 1960b)
- **theoretical** = radio evolution (Shklovsky 1960a)
- no particular reason to use Σ (except historical)

$$\Sigma_\nu = \frac{L_\nu}{\pi^2 D^2} = \frac{4\pi \varepsilon_\nu V}{\pi^2 D^2} = \frac{4\pi d^2 S_\nu}{\pi^2 D^2} = \frac{S_\nu}{\Omega_s} = \frac{4}{3} R f \varepsilon_\nu$$

$$\varepsilon_\nu = c_5 K (B \sin \Theta)^{\alpha+1} \left(\frac{\nu}{2c_1} \right)^{-\alpha}$$

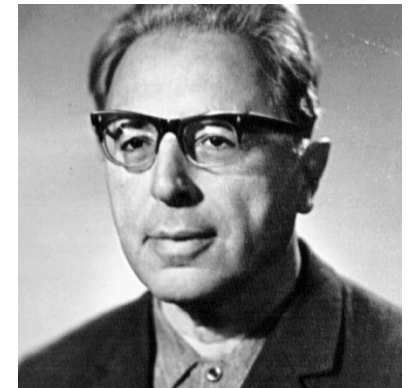
synchrotron
emissivity
..

- synchrotron nebula

$$dU = -p dV, \quad \frac{dU}{U} = \frac{dE}{E} = -\frac{1}{3} \frac{dV}{V}, \quad N(E) dE = K E^{-\Gamma} dE, \\ p = \frac{1}{3} u = \frac{1}{3} \frac{U}{V} \quad (\gamma = 4/3)$$

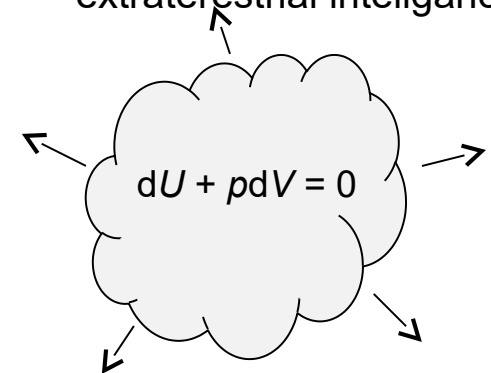
$$K \propto n E^{1-\Gamma} \propto \frac{N}{V} E^{1-\Gamma} \propto D^{-(2+\Gamma)}, \quad B \propto D^{-2}$$

$$\Sigma_\nu \propto D^{-\beta} = D^{-4(1+\alpha)} = D^{-6}, \quad \boxed{\beta = 6} \quad (\alpha = 0.5)$$



Josif Samuilovich Shklovsky
(1916 – 1985)

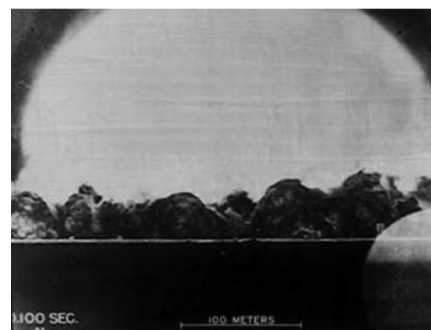
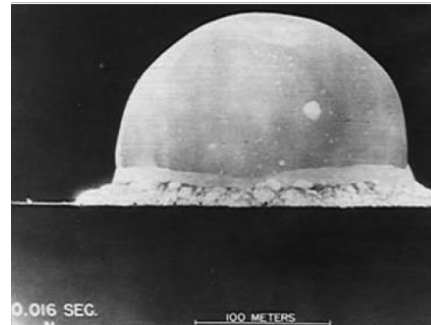
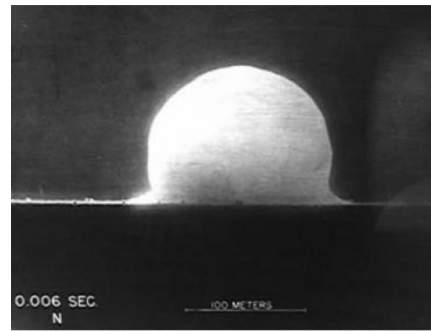
- Soviet astrophysicist, a corresponding member of Soviet Academy of Sciences
- works on Solar corona, supernovae and their remnants, cosmic rays
- extraterrestrial intelligence



- Sedov – Taylor – von Neumann blast wave solution

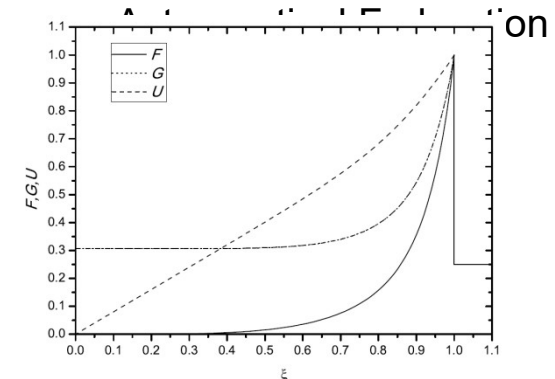
$$R = 1.15 \left(\frac{E_0}{\rho_0} \right)^{1/5} t^{2/5}$$

- analytical solution of Euler equations for a point explosion in a uniform medium (Sedov 1946, von Neumann 1947)
- numerical solution obtained independently by Taylor (1950) who analyzed old photographs and deduced and published the energy released in the Alamogordo atomic bomb probe - a classified information at that time
- Duric & Seaquist (1986) used Bell's theory and Sedov solution to deduce new theoretical $\Sigma - D$ relation



Leonid Ivanovich Sedov
(1916 – 1985)

- Soviet physicist, a member of Soviet Academy of Sciences
- the first chairman of the USSR's space exploration program, the president of the International



RADIO EVOLUTION OF SNRs

- Duric & Seaquist (1986)

- the same injection energies for p⁺ and e⁻

$$B \propto D^{-\delta}, \quad v_o = \sqrt{m_e/m_p} c \approx 7000 \text{ km/s.}$$

$$\beta = \begin{cases} 6\alpha + \delta\alpha + \delta - 1, & v \gg v_o \\ 3\alpha + \delta\alpha + \delta - 1, & v \ll v_o \end{cases}$$

$$\Sigma_\nu \propto D^{-(3\alpha + \delta\alpha + \delta - 1)} \propto D^{-7/2}$$

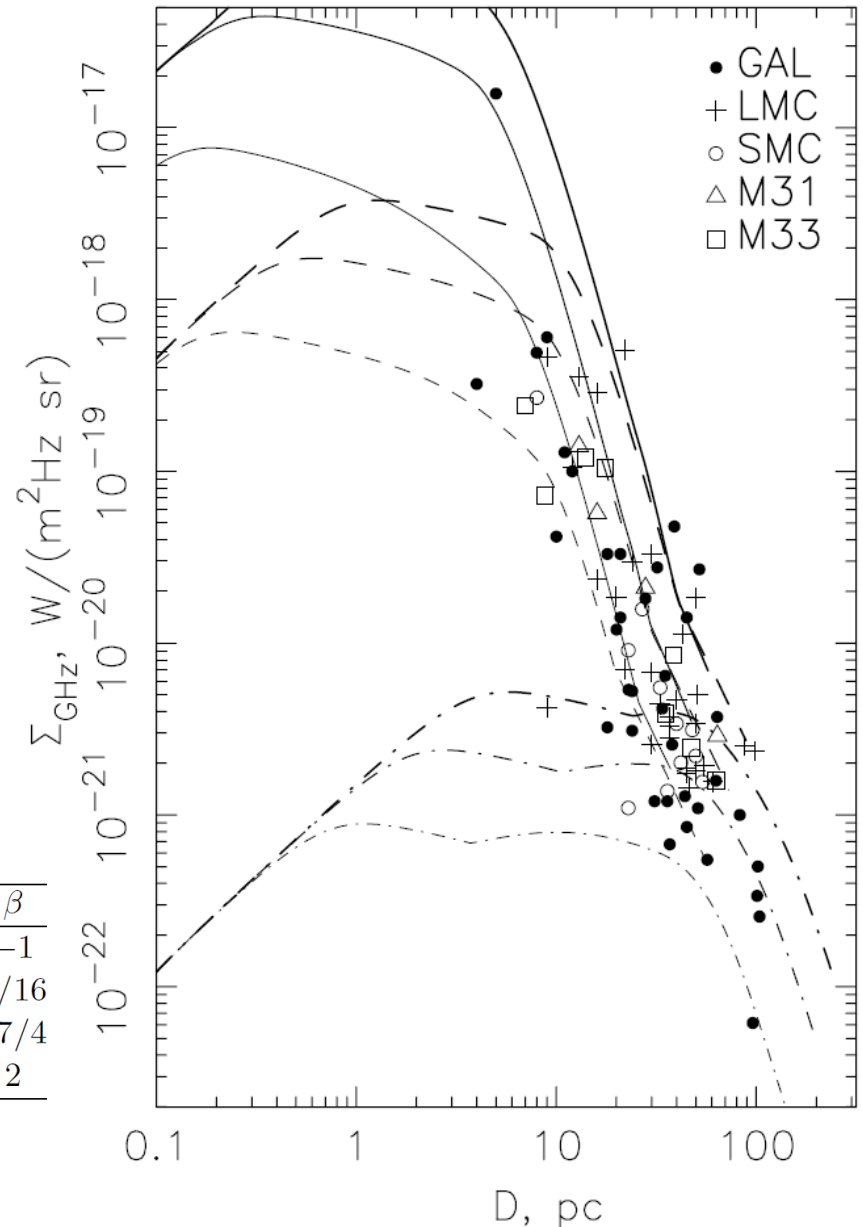
$$\boxed{\beta = 3.5} \quad (\alpha = 0.5, \delta = 2) \quad - \quad \text{late Sedov}$$

- Berezhko & Völk (2004)

- semi-analytical and numerical results (Ia)

- the same injection momenta ($K_{\text{ep}} = 10^{-2}$)

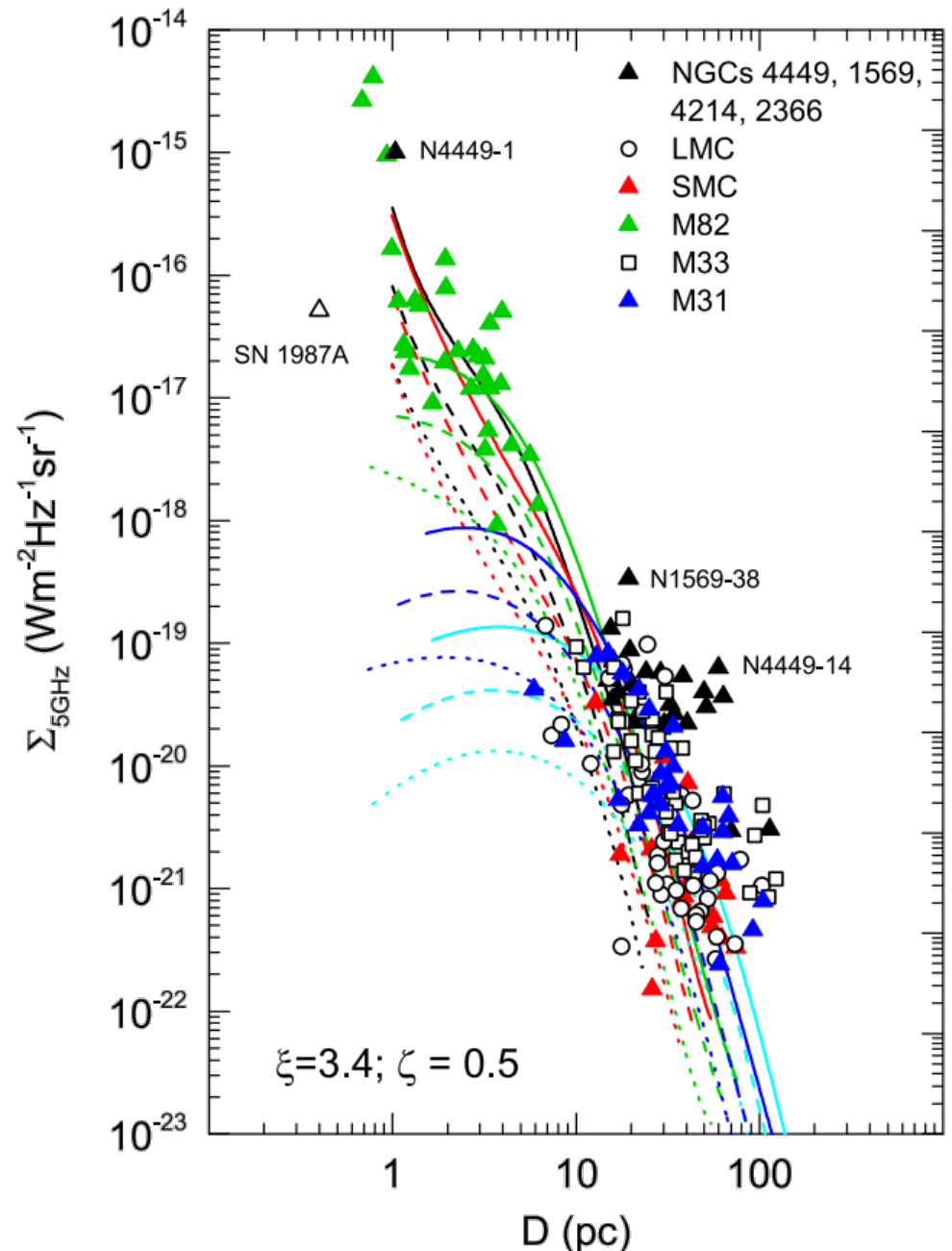
Phase	$v \propto$	$\Sigma_\nu \propto$	β
Early free expansion	$(E_o/\mathcal{M}_o)^{1/2}$	D	-1
Late free expansion	$(E_o^2/(\mathcal{M}_o\rho_o))^{1/4} D^{-3/4}$	$D^{-(1+3\alpha)/8}$	5/16
Early Sedov	$(E_o/\rho_o)^{1/2} D^{-3/2}$	$D^{-(7+3\alpha)/2}$	17/4
Late Sedov	$(E_o/\rho_o)^{1/2} D^{-3/2}$	D^{-2}	2



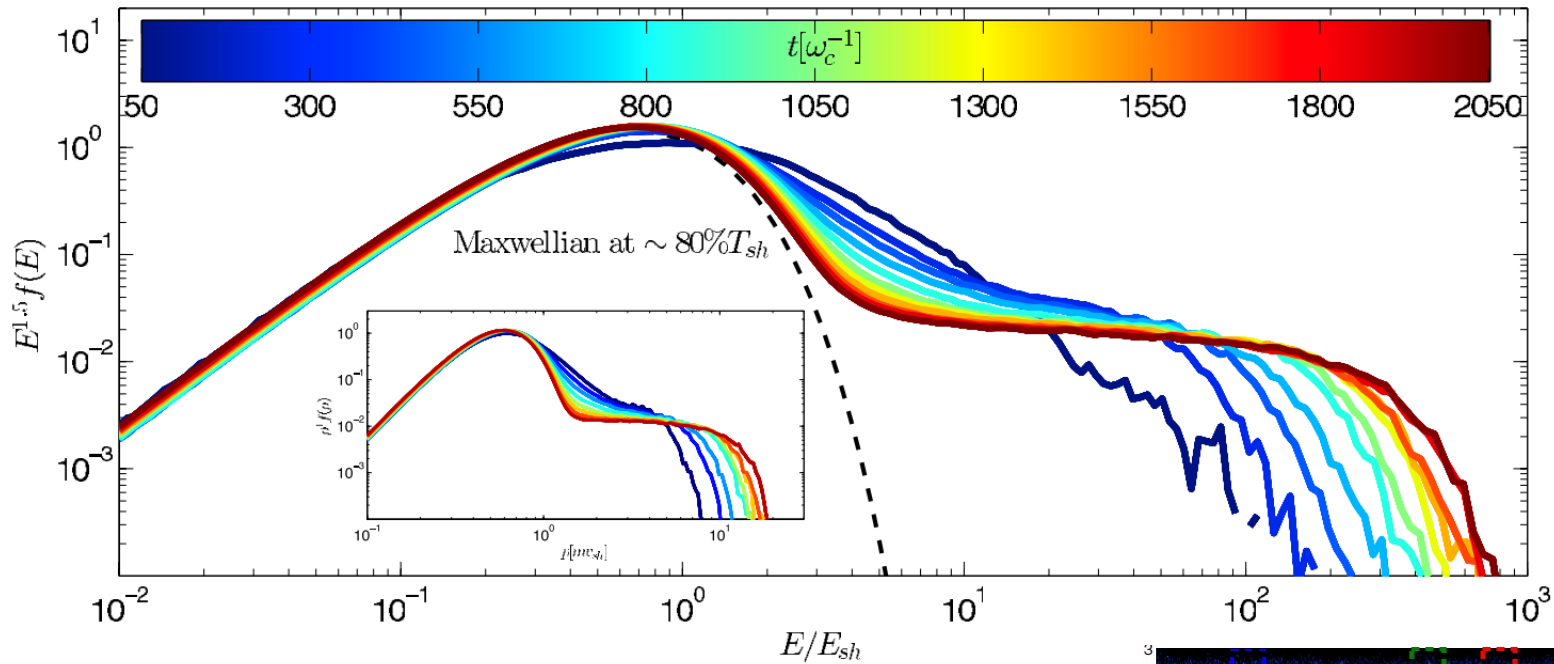
- Pavlović et al. (2018)
- combined hydrodynamic simulations with PLUTO code and particle acceleration module based on Blasi (2002a,b) semi-analytical non-linear DSA model
- the same injection momenta for p^+ and e^-
- electron to proton ratio at high energies $K_{ep} = 10^{-2}$
- evolutionary track similar to the ones obtained by Berezhko & Völk (2004) but for type Ia and II SNe

$$\rho = \frac{\dot{M}}{4\pi v_w R^2} + \rho_0$$

- parameters: ξ (higher injection parameter $\rightarrow$ stronger modification) and ζ (Alfven-heating parameter)



- PIC simulations ([Caprioli & Spitkovsky 2014](#))

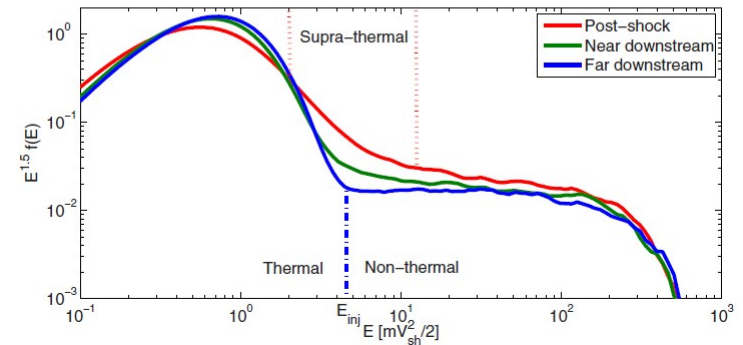
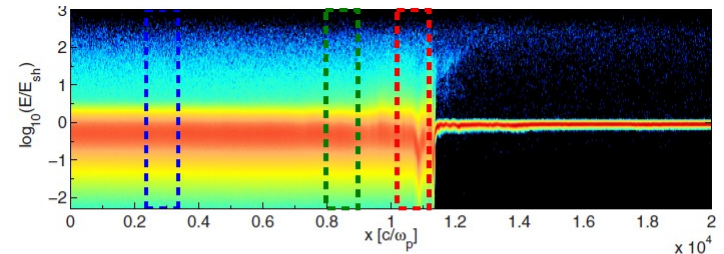


- the far downstream spectrum can be well represented by the sum of Maxwellian and a power law (a la [Bell \(1978\)](#) spectra).
- [Blasi et al. \(2005\)](#) injection recipe (for p^+ and e^-)

$$\eta = \frac{4}{3\sqrt{\pi}} (R_{\text{sub}} - 1) \xi^3 e^{-\xi^2}$$

$$\xi = p_{\text{inj}} / p_{\text{th},2}$$

injection
efficiency



- [Arbutina \(2023\)](#) prescription
- toy model - CRs in the test-particle regime (simple power law spectra)

$$f(p) = kp^{-\Gamma-2}, \quad N(E) = 4\pi p^2 f(p) = KE^{-\Gamma}$$

$$K = 4\pi k c^{\Gamma-1}, \quad k = \frac{3r}{r-1} \eta n p_{\text{inj}}^{\Gamma-1}, \quad \Gamma = \frac{r+2}{r-1}$$

- magnetic field – resonant + non-resonant (Bell's instabilities) + compression

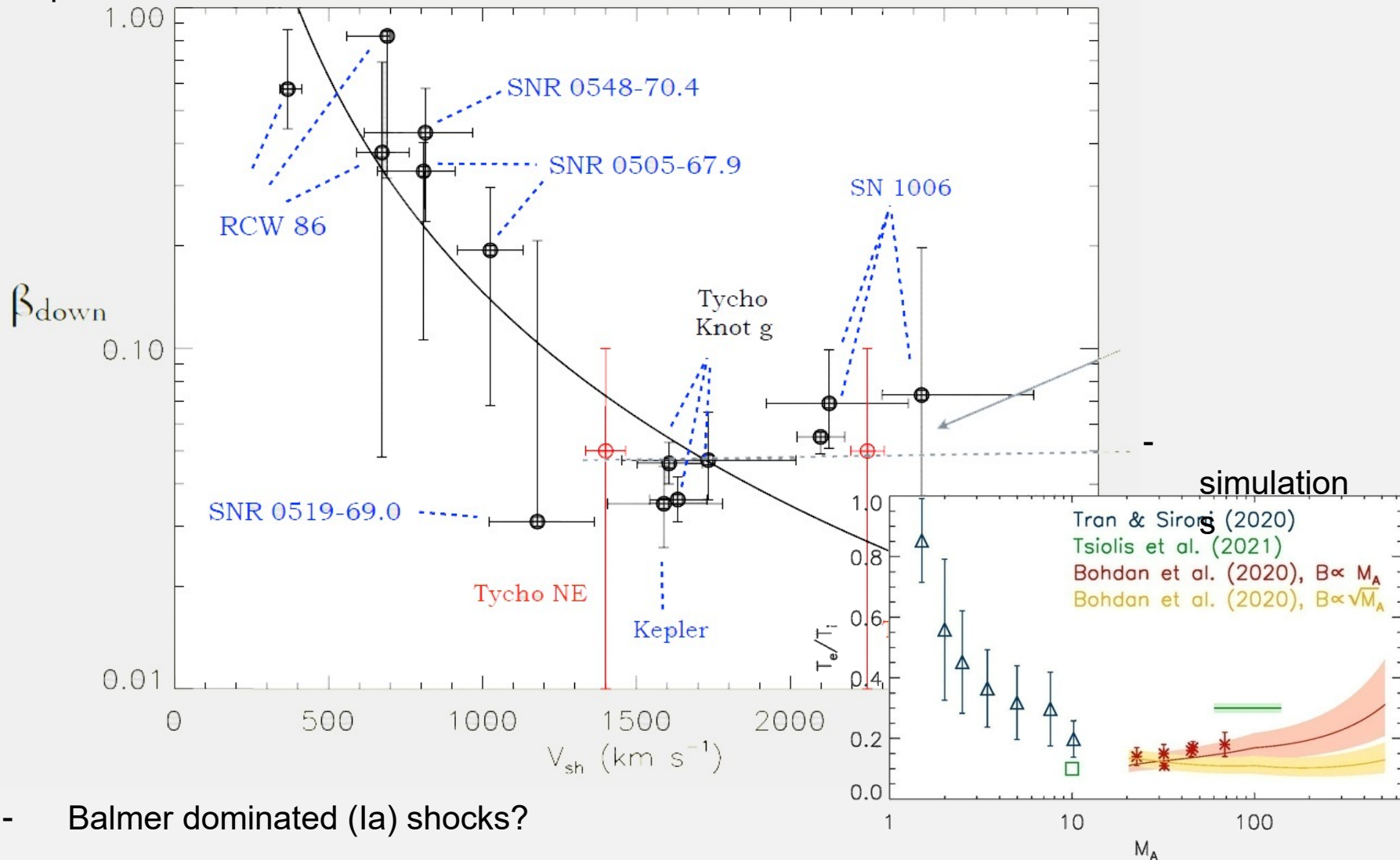
$$B^2 = 8\pi \left(r^2 \frac{1-\zeta}{2M_A} P_{\text{CR}} + \frac{1+(j-1)r^2}{j} \frac{v_s}{2} \frac{u_{\text{CR}}}{c} \right) + \frac{1+2r^2}{3} B_0^2.$$

- we can apply [Blasi et al. \(2005\)](#) recipe for electrons, and assuming $\eta_p \sim \eta_e$ get electron-to-proton ratio at high energies

$$K_{\text{ep}} = \left(\frac{p_{\text{inj}}^{\text{e}}}{p_{\text{inj}}^{\text{p}}} \right)^{\frac{3}{R_{\text{sub}}-1}} = \left(\frac{m_{\text{e}} T_2^{\text{e}}}{m_{\text{p}} T_2^{\text{p}}} \right)^{\frac{3}{2(R_{\text{sub}}-1)}} = \left(\frac{m_{\text{e}}}{m_{\text{p}}} \beta \right)^{\alpha},$$

$$\beta \approx \frac{\frac{2(\gamma-1)}{(\gamma+1)^2} m_{\text{e}} u_0^2 + \Delta E}{\frac{2(\gamma-1)}{(\gamma+1)^2} m_{\text{p}} u_0^2}, \quad (\text{Arbutina \& Zekovic 2021})$$

- the temperature ratio β depends on electron heating (Ghavamian et al. 2007, van Adelsberg et al. 2008, Raymond et al. 2023), $\Delta E \sim 0.3 \text{ keV} = \text{const}$ for lower shock speeds?



- Balmer dominated (Ia) shocks?

- dynamical formula ([Arbutina 2005, 2017, Bozzetto et al. 2017](#))

$$\frac{1}{2}v^2 = \frac{k_1 E_0}{k_2 M_0 + k_3 \frac{\dot{M}}{v_w} R + 4\pi R^3 \rho_0 / 3},$$

- parameters for type Ia, Ib/c, Ic pec and "SMB" and "SM" ISM

$$\rho = \frac{\dot{M}}{4\pi v_w R^2} + \rho_0$$

$$B = B_0 (\rho / \rho_0)^s \quad (B_0 = 5 \mu\text{G})$$

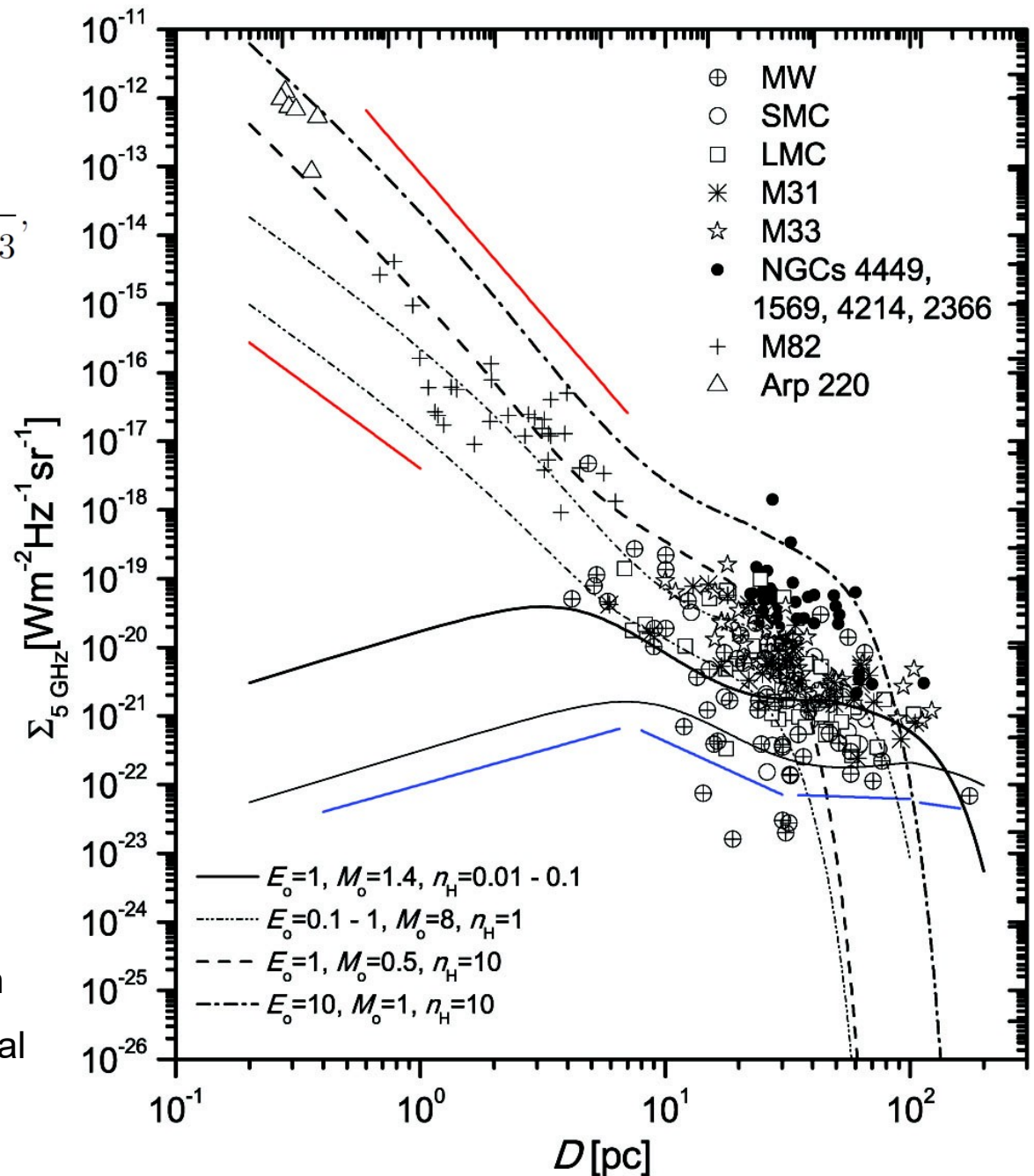
$$\dot{M} \sim 2 \times 10^{-5} M_\odot \text{yr}^{-1} \quad (\text{II})$$

$$\dot{M} \sim 1 \times 10^{-4} M_\odot \text{yr}^{-1} \quad (\text{Ib/c})$$

$$v_w \approx 10 \text{ km/s}$$

$$c_s = 10 \text{ km/s}$$

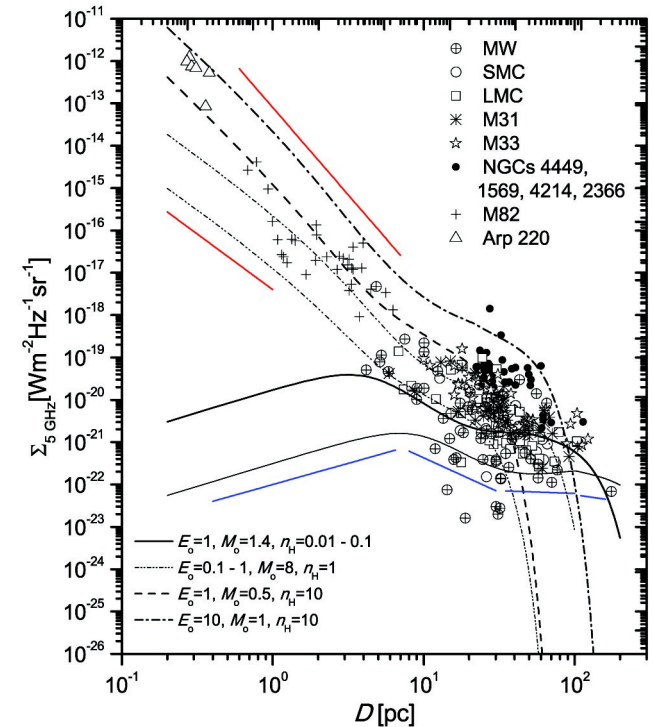
- E_0, M_0, n_0 given on the graph
- evolutionary tracks - numerical results
- [Bozzetto et al. \(2017\)](#) data compilation



- analytical results

Phase	T_e/T_p	n	v	B	α	β	=
Free expansion	const.	—	const.	$n \cdot v^{3/2}$	0.5	-1	-1
Wind interaction (Ib/c)	const.	D^{-2}	$D^{-1/2}$	$n \cdot v^{3/2}$	0.5	$11(\alpha + 1)/4$	$33/8$
Wind interaction (II)	const.	D^{-2}	$D^{-1/2}$	$n^{1/4} v^{1/2}$	0.5	$7(\alpha + 1)/4$	$21/8$
Early Sedov	const.	const.	$D^{-3/2}$	$n^{1/4} v^{1/2}$	0.5	$(15\alpha - 1)/4$	$13/8$
Mid-Sedov	$\Delta E_0 / (\frac{1}{2} m_p v^2)$	const.	$D^{-3/2}$	$n^{1/4} v^{1/2}$	0.5	$(3\alpha - 1)/4$	$1/8$
Late Sedov	1	const.	$D^{-3/2}$	$\gtrsim B_0$	> 0.5	$3\alpha - 1$	$1/2$
Evolution end	1	const.	c_s	B_0	∞	∞	∞

- other parameters: $\xi = 4$ (test particle regime), $\zeta = 0.5$, $j = 3$, $\iota = 1$.
- for simplicity $s = 0$
- evolution end -> isothermal (radiative) phase, dissipation
- CR back reaction -> shock modification -> non-linear DSA
- what do we make of all this?





- ***empirical*** relation – not particularly useful
- very likely different parameters of SN explosion and SNR evolution (ejecta mass, energy, ISM density) introduce confusion
- investigate different classes or sub-classes of SNRs?
- ***theoretical*** relation – bears her own problems
- different acceleration/injection recipe -> different results?
- need for robust theory!
- PIC simulations?

An insights from PIC simulations is primarily needed for understanding the three things:

- a) how do injection efficiencies of both protons and electrons,
- b) downstream electron-to-proton temperature ratio, and consequently
- c) CR electron-to-proton ratio at high energies, all change with shock velocity i.e. Mach's (Alfven-Mach) number.

THANK YOU!